A Comparative Study of DG Impacts on Power System Stability under Equal and Unequal Load Growth Scenarios

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Abstract— **The distributed generation (DG) technology is developed to meet the increasing load demand at a load centre by exploring locally available energy sources, including renewables. Although DG technology offers many technical and economic benefits, certain issues need further research. The impact of DG penetration on the system's stability is one such issue that requires significant attention in power system research. This study investigates the impact of DG penetration on the system's small-signal and transient stability under equal and unequal load growth scenarios. The unequal load growth condition is simulated by employing an orthogonal array (OA), and the impact of DG penetration on small-signal stability is analyzed through critical eigenvalues while time-domain indicators were employed to analyze transient stability. The importance of unequal load growth conditions while evaluating the impact of DG penetration on stability is demonstrated on a 3 generators, 9 bus WSCC power system.**

Keywords—Critical damping ratio, Distributed generation, Penetration level, Power system stability, Time-domain indicators, Unequal load growth.

I. INTRODUCTION

The concept of meeting increased power demand at a load centre by a distributed generation (DG) source employing locally available energy resources has recently attained significant attention in power system research. Environmental, regulatory, and commercial factors contributed in the development of DG technology. The main idea of DG is to explore available energy resources and generate electricity at sites close to end-user locations [1, 2].

The main objective in the development of DG technologies is to utilize the energy efficiently with reduced losses, reduced operation and maintenance cost, improved power quality, high reliability, ease of integration, and low carbon emissions [3]. Thus, it is expected that DG technologies will have a significant contribution to the growth of power systems in the near future [1, 3]. However, although DG offers many benefits, certain issues that require further research [4-10].

There are several issues of power system operation with DG sources; stability is one such important issue that requires much consideration in power system research [11]. Earlier studies indicated that when applied in small amounts, DG offers major benefits [12, 13]. On the other hand, few recent studies have shown that increased DG penetration has adverse effects on power system stability. High DG penetration can cause voltage instability and a significant reduction in transient stability margin [6]. A summary of the DG and its penetration on the stability of the stand-alone, grid-connected, and hybrid power systems is available in [14].

Most of the earlier studies focussed mainly on the impact of DG penetration on voltage and transient stabilities. The literature review indicated that the DG impact analysis on transient stability was based on time-domain indicators. However, it is important to know how small-signal stability is affected by DG penetration.

In most of the earlier reported investigations, a one-DG source-at-a-time (ODGST) approach was employed to determine the DG impact on transient stability [14, 15]. In the ODGST approach, by connecting the same technology DGs at all load centres, the DG penetration is increased in steps to evaluate the impact on stability. However, different technology DGs are employed in different load centres based on the availability of energy resource at a particular load centre. Furthermore, equal load demand growth was assumed at all DG-connected load centres in the analysis. However, in reality, the rate of load demand growth is not necessarily the same at all DG-connected load centres. Further investigations are, therefore necessary to determine the effects of equal and unequal load demand growth conditions on stability under various levels of DG penetrations.

This article presents a study on the impact of rotating type DGs on stability under equal and unequal load demand growth scenarios. The small-signal stability is analyzed by evaluating the critical damping ratio, while time-domain indicators are employed to analyze the transient performance. Further, analysis of variance (ANOVA) test is employed to determine the importance of DG sources.

II. POWER SYSTEM MODELING

A. Distributed Generation System Model and Penetration Level

Fig. 1 illustrates the distributed generation model employed in the present study. A DG is directly interfaced at a load bus, and its active power generation is set equal to increased load demand at that load bus [6, 14]. This DG power setting ensures that the power generation schedule of

main synchronous generators (SG) and the power flow patterns in the transmission network remain unchanged under steady-state conditions. This distributed generation model is simple and has all the features required to study the impact of DG penetration on main synchronous generators.
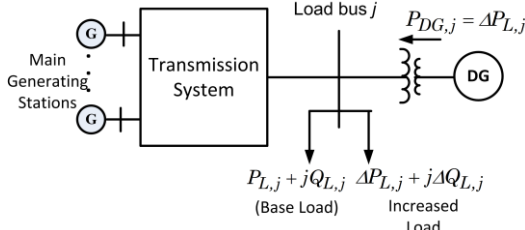

Fig. 1.   Distributed System Model

The increase in load active power is balanced by the active power generation of the DG connected at that particular load bus, as illustrated in Fig. 1. The DG penetration level (DGPL) is evaluated as a ratio of the total active DG power generation and the total load demand of the system and is given by:

$$\%DGPL = \frac{\sum_j P_{DG,j}}{P_L + \Delta P_{L,j}} \times 100 \qquad (1)$$

Where, P_L is the base case total active power demand of the system.

The fraction of load power change at a load bus-j is defined as:

$$\beta_j = \frac{\Delta P_{L,j}}{P_{L,j}} \qquad (2)$$

The active power generation of DG at load bus-j is fixed as:

$$P_{DG,j} = \Delta P_{L,j} = \beta_j \, P_{L,j} \qquad (3)$$

B.   Distributed Generation Models

In the present study, fixed-speed wind turbine generator (WTG) and SG interfaced DGs (micro turbine generator, MTG, diesel turbine generator, DTG) are considered. In deriving the models, the following assumptions are made:

- All SGs are represented by a 4th order d-q axis model equipped with IEEE type-ST1 fast-acting static exciters (modeled by a single time constant AVR with gain K_A=200 and time constant T_A=0.05 sec).

- The main SGs are equipped with two-stage fixed structure speed based lag-lead type power system stabilizers (PSS)

- The main SGs are driven by a simplified version of steam turbines [16].

- All loads are modeled as constant impedances.

The SG interfaced micro turbine (MTG) employed in the present study is based on the gas turbine model adapted from [17]. The MTG model mainly consists of turbine components in addition to a compressor and the combustor [18]. A diesel turbine governor model consists of an engine, an electric control box, and an actuator interfaced with an SG [17]. In the present study, the Type-A WTG model comprising a fixed-speed wind turbine and a direct grid-connected squirrel-cage induction generator (SCIG) is employed. A two-mass drive train representation is used in the WTG model. The WTG model assumes very fast stator transients as compared with the rotor ones. Hence, the stator flux transients are neglected in the voltage equations while deriving the fixed speed IG model suitable for stability studies [17].

The transfer function models of MTG, DTG, and the model parameters are provided in Appendix-A.

C.   Problem Formulation

The impact of DG penetration on small-stability is investigated using a critical damping ratio (ζcr). The linearized state-space model of the system and interfaced DGs, assuming constant inputs, is given by:

$$\dot{\Delta X} = A\,\Delta X \qquad (4)$$

Where A is the state matrix, and ΔX is the vector of state variables. For specified operating conditions and system parameters, eigenvalues of the entire system are determined from the state matrix A. The damping ratio associated with an oscillating mode eigenvalue $\lambda_i = \alpha_i \pm j\omega_{d,i}$ is given as:

$$\zeta_i = \frac{-\alpha_i}{\sqrt{\alpha_i^2 + \omega_{d,i}^2}} \qquad (5)$$

The critical damping ratio is

$$\zeta_{cr} = \text{Min } [\zeta \, ; \ \zeta \, \epsilon \text{ oscillating modes }] \qquad (6)$$

In the present work, the linearized state-space model is formulated in the SIMULINK/MATLAB environment [19]. In this approach, the solution of all state equations of various power system components is formulated by developing SIMULINK models. In addition, a non-iterative AC network solution is developed to update the non-state variables of the system during the simulation process.

All loads are represented by equivalent shunt admittances, and the bus admittance matrix Y_{BUS} is then updated to consider these equivalent load admittances. Hence, the current injection at load buses will be zero. The non-iterative network voltage solution can be derived from the reduced network (retaining only the generators), and is given as:

$$V = [T - Z_M T Y_N]^{-1} E \qquad (7)$$

Where Y_N is the ($2n_g$x$2n_g$) matrix of real and imaginary part of Y_{BUS} elements, n_g is the total number of generators; T is the block diagonal transformation matrix. For i^{th} SG, $T_i = \begin{bmatrix} Cos\delta_i & Sin\delta_i \\ -Sin\delta_i & Cos\delta_i \end{bmatrix}$ and δ_i=0 for SCIG. Z_M is the block diagonal matrix of machine impedance, where

$$z_{m,i} = \begin{bmatrix} -R_{a,i} & x'_{d,i} \\ -x'_{q,i} & -R_{a,i} \end{bmatrix} \quad \text{for the } i^{th} \text{ SG and}$$

$$z_{m,i} = \begin{bmatrix} -R_{s,i} & X'_{s,i} \\ -X'_{s,i} & -R_{s,i} \end{bmatrix} \text{ for the } i^{th} \text{ SCIG.}$$

$E=[E_{q1}' E_{d1}', \ldots \ldots E_{qm}' E_{dm}']^t$

A MATLAB function is developed to implement the AC network solution and embedded in the SIMULINK model. The main advantage of the SIMULINK power system model is that both state matrix and time-domain simulation results can be obtained from the same SIMULINK model.

III. CASE STUDY AND SIMULATION RESULTS

A. Test System and Simulation Setup

In the present study, a 9 bus, 3 generators WSCC benchmark power system is employed. All machine and line data are taken from [20]. The system consists of three load centres, and the total load demand is 530 MW and 200 MVAr. MTG, DTG, and WTG are interfaced at load buses 5, 6, and 8, respectively, via a transformer of (0+j0.056) pu impedance.
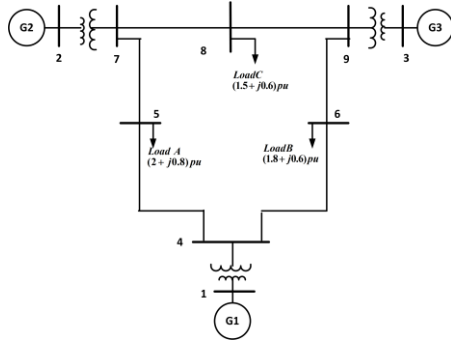

Fig. 2. WSCC Test power system

The active and reactive power at load buses 5, 6, and 8 is increased, and the active power generation of corresponding DGs is set as per Eq.(3). At any load bus, the maximum load increase is assumed as 50% of the base-load (β_j=0.5) at that bus. Therefore, as per the base-case load data, the maximum DG penetration can be calculated as 33.33%.

The unequal load growth scenario is simulated by assuming 5 discrete levels of a fraction of load change in the range [0.0:0.5] as summarized in Table I.

TABLE I.   LEVELS OF A FRACTION OF LOAD CHANGES

B_j	Levels of fraction of a load change				
	Level-1	Level-2	Level-3	Level-4	Level-5
β_{MTG}	0.02	0.14	0.26	0.38	0.5
β_{DTG}	0.02	0.14	0.26	0.38	0.5
β_{WTG}	0.02	0.14	0.26	0.38	0.5

The simulation of unequal load growth scenario requires all 125 combinations of different levels of β_j as per full factorial design (FFD), which is time-consuming. However, this problem can be overcome by employing an orthogonal array (OA) with only a few combinations of β_j levels. For 3 DG groups, each defined with 5 discrete levels of β values, the minimum number of entries required in an OA is given as:

$$N_{min} = (L-1)F + 1 \qquad (8)$$

In Eq. (8), L is the number of levels (=5), and F is the number of control factors (=3). Therefore, an appropriate 5-level OA is to be selected such that the number of entries $N > N_{min} = 13$. Hence L_{25} or 5^3 OA is selected with 25 entries. Table II summarizes the various combinations of β_{MTG}, β_{DTG},

and β_{WTG} levels as per L_{25} OA and the corresponding DGPL levels.

TABLE II. L_{25} ORTHOGONAL ARRAY AND DG PENETRATION LEVELS

En try No	Fraction of load changes			
	β_{MTG}	β_{DTG}	β_{WTG}	DGPL%
1	0.02	0.02	0.02	1.9608
2	0.02	0.14	0.14	8.6522
3	0.14	0.02	0.26	14.4885
4	0.02	0.26	0.26	19.6239
5	0.26	0.14	0.02	24.1774
6	0.14	0.14	0.38	9.0285
7	0.14	0.5	0.14	14.8184
8	0.14	0.38	0.02	22.7180
9	0.02	0.38	0.38	24.4368
10	0.38	0.02	0.14	18.5868
11	0.26	0.02	0.5	15.1457
12	0.26	0.38	0.26	20.2048
13	0.26	0.26	0.14	24.6945
14	0.14	0.26	0.5	18.8858
15	0.5	0.26	0.02	23.5209
16	0.38	0.26	0.38	20.4920
17	0.02	0.5	0.5	24.9504
18	0.38	0.14	0.26	19.1827
19	0.38	0.5	0.02	23.7849
20	0.26	0.5	0.38	27.8912
21	0.5	0.14	0.5	25.2046
22	0.5	0.02	0.38	19.4774
23	0.5	0.38	0.14	24.0470
24	0.38	0.38	0.5	28.1258
25	0.5	0.5	0.26	31.7889

The important properties and advantages of OA are summarized below:

The columns of OA are mutually orthogonal and exhibit balancing properties. As seen from Table II, all combinations of factor levels occur in an equal number of times for any pair of columns. Because of this property, an OA defines an experimental region of factors to be studied.

OA exhibits fractional factorial characteristics. Moreover, the results obtained using OA are almost the same as that can be obtained from FFD. Hence the use of OA saves computational time significantly.

Impact Study on Small-Signal Stability

Critical eigenvalues under unequal load growth scenario (UELG) are computed as per the L_{25} OA using the linearized state matrix A. For comparison purposes, for the same DG penetration levels as in Table II, critical eigenvalues are also computed under an equal load growth scenario (ELG). The value of β_j for equal load growth is determined as:

$$\beta_{MTG} = \beta_{DTG} = \beta_{WTG} = \frac{DGPL}{1 - DGPL} \qquad (9)$$

Fig. 3 depicts the comparison of ζ_{cr} under ELG and UELG scenarios.
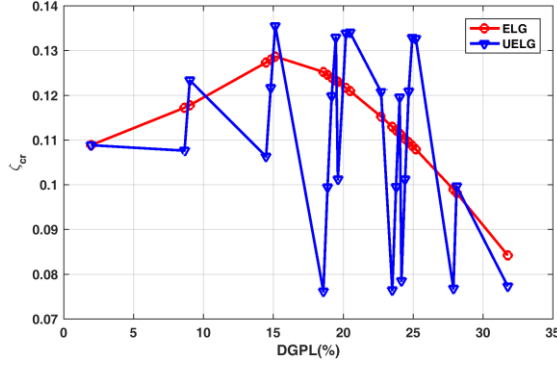

Fig. 3. Comparison of Critical damping ratio under equal and unequal load growth scenarios.

It is clear from Fig. 3 that the variations of ζ_{cr} with DGPL under UELG are quite different than under ELG conditions. Up to 15% DGPL, ζ_{cr} show an increasing tendency and as DGPL increases beyond this, there is a sharp reduction in ζ_{cr} values.

For the same DGPL, the ζ_{cr} value under UELG is significantly different than under ELG condition. The ζ_{cr} value and hence the small-signal stability depend on the active power generation of DGs. For the same DGPL, DGs' active power generation settings are different under ELG and UELG conditions and hence affect the small-signal stability. It is worth noting here that for few combinations of β values, unequal load growth conditions resulted in much higher values of ζ_{cr} as compared to equal load growth conditions.

ANOVA test was performed to determine the relative importance of each DG source and their interaction effects on the variance of ζ_{cr} values. ANOVA test is performed using the "anovan" routine available in the MATLAB toolbox, which gives the percentage contribution of each DG source (direct or main effects) and the contribution due to the interaction. The results of ANOVA are summarized in Table III.

TABLE III.   ANOVA TEST RESULTS

DG Source	% Contribution
Direct effect of MTG	6.72
Direct effect of DTG	86.75
Direct effect of WTG	0.02
Interaction between MTG and DTG	6.33
Interaction between DTG and WTG	0.14

The ANOVA test results indicate that the DTG alone contributes around 86.75% towards the variance of ζ_{cr} values. There is a small interaction effect (6.33%) between MTG and DTG, while all other interactions are negligible. The direct or main effect indicates the overall effect of each DG source alone to change the critical damping ratio, while the interaction effect indicates the mutual dependency of the two DG sources. The ANOVA test results are validated by the analysis of means (ANOM), as illustrated in Fig. 4.
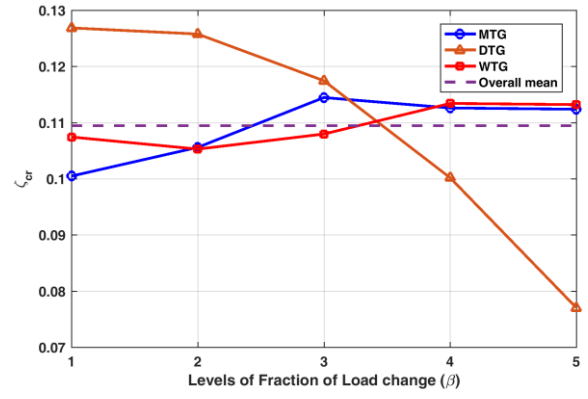

Fig. 4. Results of Analysis of means (ANOM)

In Fig. 4, the overall mean value of ζ_{cr} (0.1095) is shown by dotted line. It is clear that a change in the level of β of DTG results in large variations of mean ζ_{cr} around the overall mean and therefore contributes significantly towards the variance. Moreover, it is seen that the mean value of ζ_{cr} reduces sharply when the level of β of DTG is increased. In the case of MTG, there is a slight increase in the mean value of ζ_{cr} when β increased up to level 3.

B. Impact study on Transient Stability

The comparison of relative slip deviation response ($S_{m,21}$ and $S_{m,31}$) of main SG under ELG and UELG conditions and DGPL=24.95% for 125ms duration 3-phase self-clearing fault on bus 9 is depicted in Fig. 5. The corresponding transient response comparisons for MTG, DTG, and WTG for this fault are shown in Fig. 6. At DGPL=24.95% the values of fraction of load changes for UELG are β_{MTG}=0.02, β_{DTG}=0.5, and β_{WTG}=0.5 while, for ELG it is β_{MTG}= β_{DTG}= β_{WTG}= 0.3325.

It is seen from Fig. 5 and Fig.6 that the transient response of main SG and all DGs are more oscillatory under UELG as compared to ELG. Since under UELG condition, the DTG active power generation is set at maximum (0.5), the system's damping is very low, resulting in poor transient performance.
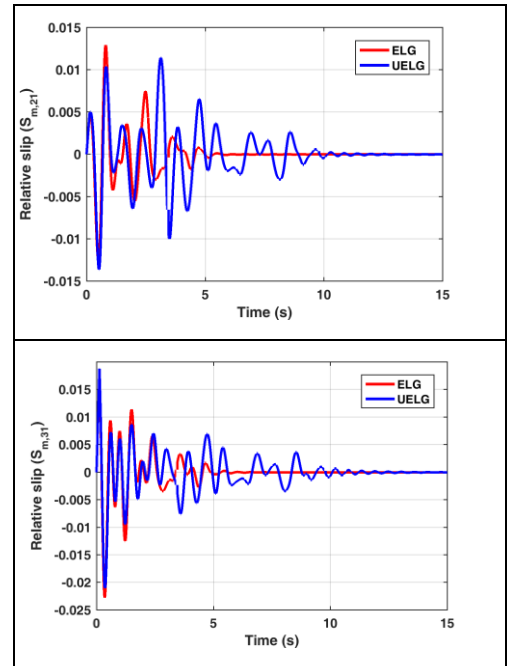

Fig. 5. Transient response comparison of main SG under ELG and UELG (DGPL=24.95%)

In order to quantify the impact of DGPL on system transient response under ELG and UELG, two time-domain indicators, namely maximum slip deviation (MSD) and settling (ST) are employed in this study. The MSD value indicates the worst value of maximum deviation among a group of responses. The settling time is measured using a 2% tolerance limit.

It is observed from Fig. 7 that the MSD values decrease as DGPL is increased in the case of main SG, MTG, and DTG, whereas MSD of WTG increases with an increase in DGPL. It is also seen that MSD values are significantly different under ELG and UELG scenarios. For few DGPL values, unequal load growth results in much larger MSD values than equal load growth condition.

From Fig. 8, it is seen that the settling time remains almost same for small DG penetration levels (DGPL<15%). However, as DGPL increases beyond 15%, the settling times of main SG and all DGs tend to increase, indicating growing oscillations. It is also observed that for the same DGPL, the settling time depends on the type of load growth, equal or unequal.
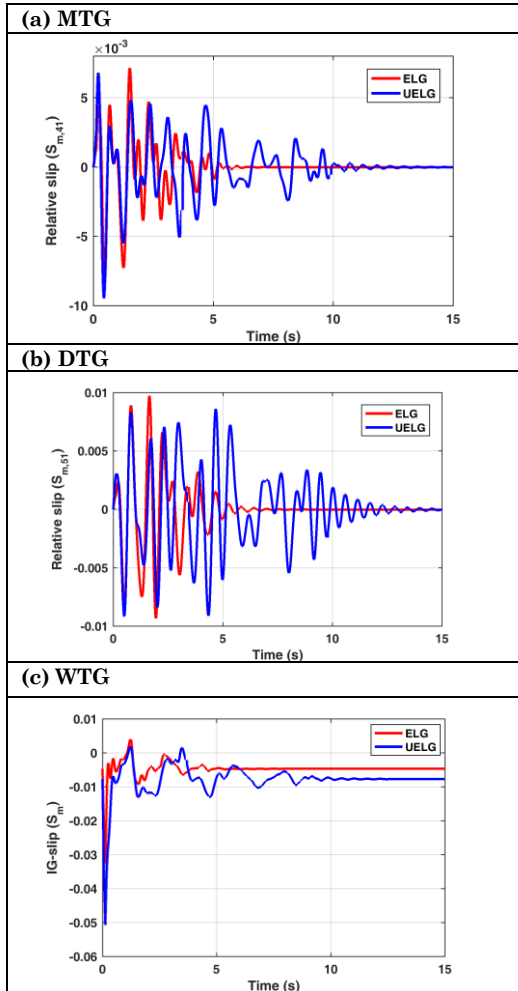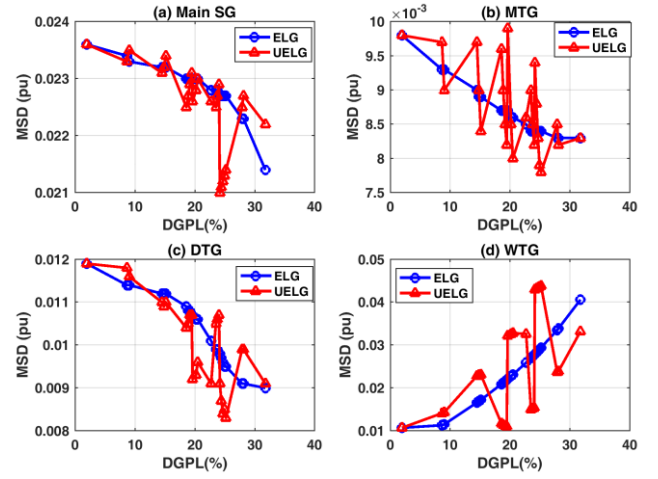

Fig. 7. Comparison of maximum slip deviation (MSD) under ELG and UELG conditions
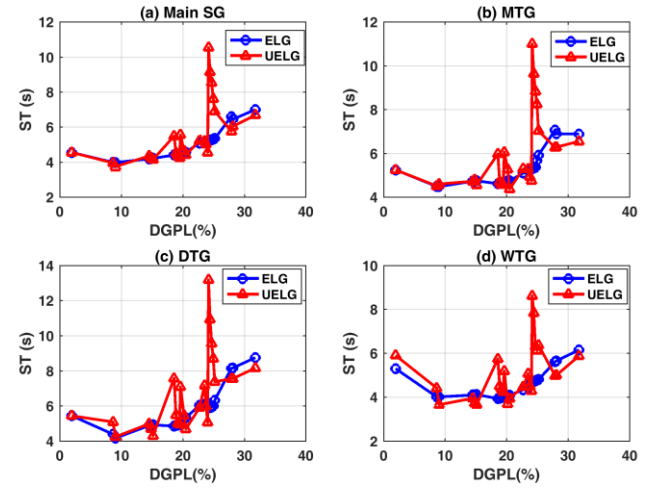

Fig. 8. Comparison of settling time (ST) under ELG and UELG conditions

From the above discussion, it is clear that the small-signal and transient stability of the power system depends not only on the DG penetration but also on the type of load growth scenario in the system. For the same DG penetration level, the stability performance may be significantly different under equal and unequal load growth scenarios. The definition of DG penetration as given by Eq. (1) does not reflect the type of load growth scenario. Further investigations are, therefore, necessary to arrive at a new definition of DG penetration that reflects both load growth and type of load growth, equal or unequal.

IV. CONCLUSIONS

This paper provided a comprehensive analysis of the impact of DG on power system stability under equal and unequal load growth scenarios. SG interfaced DG sources (MTG and DTG) and fixed-speed WTG IS employed for this purpose. The study revealed that the power system exhibited different stability performances under equal and unequal load growth scenarios for the same DG penetration level.

Fig. 6. Transient response comparison under ELG and UELG of (a) MTG (b) DTG (c) WTG (DGPL=24.95%)

APPENDIX-A

MODELS OF DIESEL AND MICRO TURBINES AND MODEL PARAMETERS

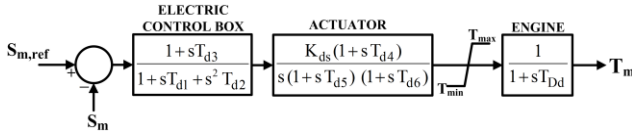(All data are in per-unit unless specified otherwise)

Fig. A1 Diesel turbine model

T_{d1}=0.2s; T_{d2}=0.02s; T_{d3}=0.2s; T_{d4}=0.25s; T_{d5}=0.009s; T_{d6}=0.0384s; T_{Dd}=0.02s; K_{ds}=30.0.
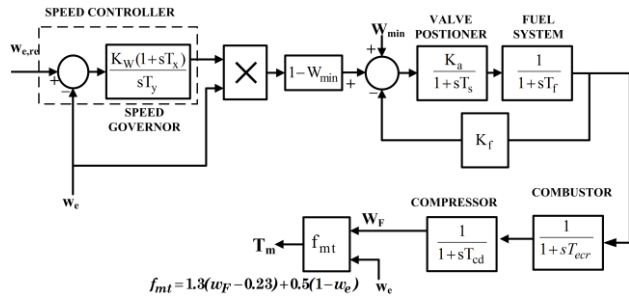

$$f_{mt} = 1.3(w_F - 0.23) + 0.5(1 - w_e)$$

Fig. A2 Micro- turbine model

K_f=0.0; K_w=16.7; K_a=1.0; T_s=0.05s; T_x=0.0; T_y=1.0s; T_f=0.4s; T_{cd}=0.2s; T_{ecr}=0.01s.

APPENDIX-B

DYNAMIC MODEL EQUATIONS SYNCHRONOUS AND INDUCTION GENERATORS

(a) Synchronous Generator

$$\frac{d\delta}{dt} = \omega_b S_m$$

$$\frac{dS_m}{dt} = \frac{1}{2H}[T_m - T_e]$$

$$\frac{dE'_q}{dt} = \frac{1}{T'_{do}}[-E'_q + (x_d - x'_d)i_d + E_{fd}]$$

$$\frac{dE'_q}{dt} = \frac{1}{T'_{qo}}[-E'_d - (x_q - x'_q)i_q]$$

$$\frac{dE_{fd}}{dt} = \frac{1}{T_A}[K_A(V_{ref} - V_t + V_{pss}) - E_{fd}]$$

(b) Induction Generator

$$\frac{d\gamma}{dt} = 2\pi f[\omega_m - \frac{\omega_G}{N_G}]$$

$$\frac{d\omega_m}{dt} = \frac{1}{2H_m}[T_{ae} - K_s\gamma - D_m\omega_m]$$

$$\frac{d\omega_G}{dt} = \frac{1}{2H_G}[T_m - T_e - D_G\omega_G]$$

$$\frac{dE'_q}{dt} = \frac{-1}{T'_0}[E'_q - (X_s - X'_s)i_d] + S_m\omega_s E'_d$$

$$\frac{dE'_d}{dt} = \frac{-1}{T'_0}[E'_d - (X_s - X'_s)i_q] - S_m\omega_s E'_q$$

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