Asian Journal of Convergence in Technology Volume II, Issue III
Issn No.:2350-1146, I.F-2.71

Performance analysis of Coded DFT-OFDM and wavelet based OFDM Systems

Atul M Kulkarni,
Dr.BATU,IOPE,
Lonere,maharashtra
atulworld@gmail.com

Dr.Prashantha Kumar H,
NITK Surthkal
Karnataka
hprashanthakumar@gmail.com

Kumar D
SJCIT Banglore
Karnataka
kumard243@gmail.com

Abstract— **To cope with the requirements of higher bandwidth efficiency wavelet transform based OFDM system (WOFDM) have been proposed in place of Fast Fourier transform based OFDM systems because of its high spectral containment by many researchers. *This paper gives the comparative analysis to check the performance of conventional OFDM and Wavelet based OFDM with different channel coding techniques with different code rate/parameters over the Rayleigh fading channel. The BER performance of comparative analysis shows that Wavelet based OFDM system gives better coding gain than conventional OFDM system and with various forward error correction techniques the performance is further enhanced.***

***Index Terms*— BCH, BER, Coding gain, Convolutional Codes, OFDM, RS , Wavelet based OFDM(WOFDM)**

I. Introduction

TO meet the current requirement of high data rate transmission with efficient bandwidth utilization and lower bit error rate coded OFDM is the solution. A high data rate transmitted signal has a large bandwidth, this means that it is subjected to frequency selective fading, which can distort the signal significantly. One solution is to parallely transmit the data symbols on many narrow band sub-channels by dividing the available bandwidth for transmission. As low data rate sub channel's frequency component encounter an almost flat channel and this is achieved through OFDM scheme.

ORTHOGONAL FREQUENCY DIVISION MULTIP-LEXING is a FFT based multicarrier modulation (MCM) scheme, in which higher data rate serial input symbols is splited or converted to multiple parallel lower rate symbols, and each is carried on separate orthogonal subcarriers, orthogonal subcarrier refers to maintaining the each subcarrier's value as zero exactly at the center frequency of every other subcarrier. Because of the orthogonality and the overlapping nature of subbands OFDM system does not require intercarrier guard band, but the cyclic prefix is added during the initial part of OFDM symbol as a guard band to solve the problem of inter symbol interference (ISI) due to nearby OFDM symbols [1],[2].

Overlapping nature of sub-bands makes OFDM system bandwidth efficient also, it`s need to make sure that coherence bandwidth of channel should be more than the bandwidth of each subcarrier, which makes the subcarriers to experience flat fading, hence no inter symbol interference, if the cyclic prefix i.e. guard interval is made greater than the channel`s delay spread, ISI can be completely avoided. Because of this numerous advantages OFDM is employed in all domains of wireless communications. RS and LDPC coding are used with conventional OFDM for (Digital Video Broadcasting - Satellite) DVB S and DVB S2, WiMAX respectively [3],[6].

OFDM has some demerits, because of the multipath fading channel inter-symbol interference ISI occurs, and this effect can be nullified completely with the help of cyclic prefix (CP), but this reduces spectral efficiency. The OFDM transmitted signal exabits high peak values in the time domain since many components of the subcarrier gets added because of an IFFT operation. That's why OFDM systems well known to possess high peak to average power ratio. In case of Time variant channels orthogonality among subcarriers gets affected due to doppler shift, causes carrier frequency offset [2,3,4,5].

The wavelets based OFDM systems are further enhances spectral efficiency as appending cyclic prefix at the start of OFDM symbol is not required, and possess very narrow side lobes hence outperforms conventional OFDM systems. WOFDM with coding techniques and different wavelet basis further enhances the performance of the system. The IEEE 1901-2010 standard based on wavelet OFDM "Wavelet PHY" includes a forward error correction techniques concatenated Reed-Solomon and convolutional code, and also provides option for low density parity check (LDPC) coding [6],[8],-[16],[17].

This paper gives the comparative analysis for the performance of coded WOFDM and coded conventional OFDM. Forward error correction (FEC) is the process of adding structured redundant bits to the 'k' message bits so as to get 'n' bits (where n-k bits referred as check bits), this is so achieved that received data at the receiver can be corrected if error is occurred during the transmission. Here performance of WOFDM and DFT-OFDM is compared, using BCH, RS and convolutional coding techniques, over AWGN and Rayleigh channel. Section II provides an insights into the fundamentals of conventional OFDM systems, while section III gives description of wavelet based OFDM system model. In section IV the performance analysis is discussed with different simulation results, and section V conclusions.

II. FFT Based OFDM System

The OFDM signals can be generated by a bank of traditional multiplier-filter modulators and demodulated by a bank of traditional correlatores however, it can be generated more efficiently by an N point IFFT and demodulated by an N point FFT. It is because of this that OFDM has found widespread uses in all types of communication systems. The block diagram of FFT/DWT based OFDM transceiver with coding shown in Fig. 1.

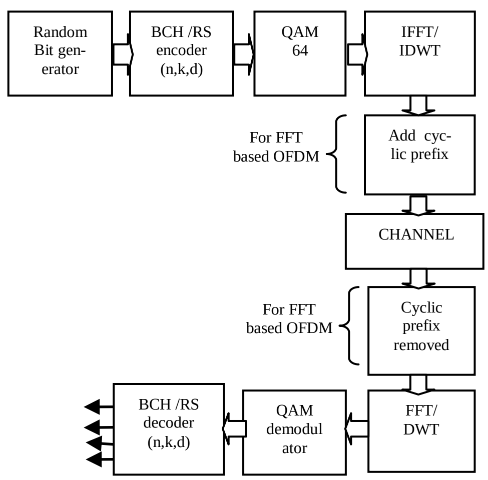


Fig. 1. FFT and wavelet based OFDM system model.

OFDM transmitter after channel coding maps message bits to QAM symbols (for simulation we used 64 QAM). These QAM symbols then divided to parallel streams, all the 'N' symbols ('N'-No.of subcarriers/IFFT length, for simulation we used 64 subcarriers) after serial to parallel conversion transmitted on different subcarriers. Let`s For example. X_l (k) represents the ith transmitted symbol on the kth sub-carrier, $i = 0,1,2, \dots \dots \infty, k = 0,1,2, \dots \dots N-1.$ Due to this the transmission time of symbols is increased to NT_s , where T_s is one QAM symbol period and N is number of subcarriers, which gives length of one OFDM symbol to T_{sym} (i.e., $T_{sym} = NT_s$). The orthogonality between two neighboring carriers in the spectrum is maintained as the carriers are spaced $1/T_s$ apart. Let $\varphi_{i,k}(t)$ defines the ith OFDM signal on the kth sub-carrier, which is given as

$$\varphi_{i,k}(t) = \begin{cases} e^{j2\pi f_k (t-T_{sym})}, & 0 < t \le T_{sym} \\ 0, & elsewhere \end{cases} \quad \dots\dots.(1)$$

In the continuous time domain the base-band and pass-band OFDM signals can be expressed mathematically as

$$x_i(t) = Re\left\{\frac{1}{T_{sym}} \sum_{l=0}^{\infty} \left\{ \sum_{K=0}^{N-1} X_i[k]\, \varphi_{i,k}(t) \right\}\right\}$$

and

$$x_i(t) = \sum_{l=0}^{\infty} \sum_{k=0}^{N-1} X_i[k] e^{j2\pi f_k (t - iT_{sym})} \quad \dots \dots \dots (2)$$

The discrete time OFDM symbol can be calculated from the continuous time base-band OFDM signal in Equation (2) by sampling at $t = iT_{sym} + nT_s$ and $T_s = T_{sym}/N$ and $f_k = k/T_{sym}$,hence the related Discrete time OFDM symbol is

$$x_i[n] = \sum_{k=0}^{N-1} X_i[k] e^{j2\pi kn/N} \quad for\ n = 0,1,.,N-1 \quad (3)$$

Note that equation '3' is nothing but the N-point IDFT of QAM data symbols $\{X_i[k]\}_{k=0}^{N-1}$ and with the help of IFFT (inverse fast fourier transform) algorithm we can compute it efficiently. This fact implies that samples of the complex envelope of an OFDM signal can be generated by IDFT, and at receiver the $X_i[k]$ reconstructed by means of appling DFT [3].To minimize the effect of inter symbol interference (ISI), a guard interval (C/P) is inserted between OFDM symbols, if the guard interval is more than the length of the channel`s delay spread, ISI can be completely avoided [6].

III. Wavelet Based Ofdm Systems

In 2006 patent is filed for a new technique where wavelet transforms is used for data symbols modulation onto a number of subcarriers by Jain and Myers, which is referred as orthogonal wavelet division multiplex, and this could be an efficient option to conventional OFDM (where we uses Fourier transforms) has been shown by Linfoot and Ibrahim [8]. As no requirement of cyclic prefix almost 20% Bandwidth can be saved.

A. Wavelets

To analyse the time variant signal in frequency domain,we can use Fourier transform with proper window size to get the accurate information if the signal is slowly varying, however if there is fast fluctuations of the signal ,where we need to find how frequently signal is varying (i.e. frequency domain information) where Fourier transform is not practical to get both time and frequency domain information simultaneously. for example, Audio signal analysis, where both time and frequency analysis is necessary.

Most common techniques that used for time frequency analysis are, the short time Fourier transform, where we moves the window over the signal which is in time domain, but because of fixed window size problem of resolution occurs., therefore solution is wavelet transform. or the wavelet packet Transform. The wavelet transform represents a time variant signal over a time by a two dimensional function of '*m*' and 'k' (where 'k' is called shifting/translation parameter and 'm' the scaling factor of the wavelet function, like Haar, Daubechies etc.).The CWT (continuous wavelet transform) of a signal *c(t)* can be defined as [10]:

$$CWT_{(m,k)} = \frac{1}{\sqrt{m}} \int c(t) \cdot \varphi\left(\frac{t-k}{m}\right) dt \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

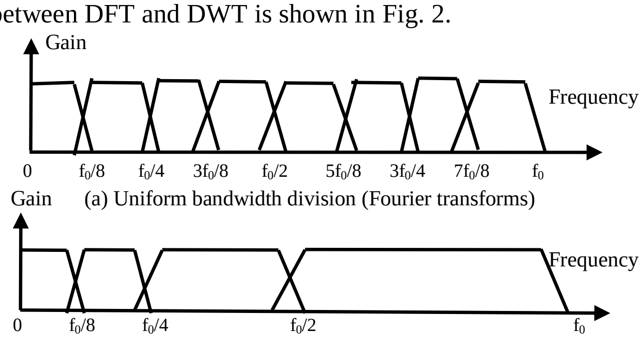
where t is the time, $\varphi(t)$ is referred as mother/basis wavelet and $\frac{1}{\sqrt{m}}\varphi\left(\frac{t-k}{m}\right)$ is the baby wavelet[10] which is a formed by either dilation or compacting the mother wavelet. There are, varieties of different baby wavelets for example symlet, morlet where the Haar wavelet can be represented as ;

$$\varphi(t) = \begin{cases} 1, & 0 \le t \le 1/2 \\ -1, & 1/2 \le t \le 1 \\ 0, & otherwise \end{cases}$$

And its scaling function is $\emptyset(t) = \begin{cases} 1, & 0 \le t \le 1 \\ 0, & otherwise \end{cases}$

Now from the theory of wavelet the any arbitrary signal is expressed to a summation of wavelet and scaling functions, and which is referred as wavelet transform (WT). It is not practical to find CWT so the discrete wavelet transform should be used. for this, a discrete wavelet is correlated with discrete signal [13].

Mallat S.G. [11] demonstrated the simplest way to realize the discrete wavelet transform from continuous wavelet transform with the help of subband filtering, where the banks of matched high pass and low pass filters are utilized. And observing for a 2^n point DFT, the uniform division of bandwidth takes place, whereas for an n-level discrete wavelet transform, the bandwidth gets logarithmically divided, here at each level low pass filter outputs i.e. half of the spectrum are decomposed. The bandwidth division structure comparison between DFT and DWT is shown in Fig. 2.



(b) Logarithmic bandwidth division (Wavelet transforms)

Fig. 2.

Complex exponentials of the Fourier transform are replaced by means of wavelet carriers with different scaling factor 'm' and translations 'k' on the time axis for wavelet transform. These wavelet carriers/functions are defined by the shifting and scaling of a unique function, called " mother wavelets " and represented by $\varphi(t)$, and importantly the orthogonality among these wavelet carriers depends on shifting/translation 'k' and scale index 'm'. The orthogonality is accomplished by generating baby wavelets for particular mother wavelet family, according to equation,

$$\varphi_{(m,k)}(t) = 2^{-m/2}\varphi(2^{-m}t - k)$$

Perfect reconstruction property of wavelet analysis actually helps to reconstruct the transmitted signal at receiver accurately. In this work the 'haar' wavelet, is considered to check the performance of BCH/RS/convolutional coded wavelet OFDM with conventional OFDM over wireless channel AWGN and Rayleigh.

B. *Wavelet Packet Transform (WPT)*

The MCM systems such as OFDM where uniform division of bandwidth utilized among subcarriers, hence wavelet transform which provides logarithmic division of the bandwidth is not suited, however the generalization of wavelet transform is wavelet packet transform, where the orthogonal carriers are "wavelet packets". ↓2 refers to the two times down sampling, In case of discrete wavelet transform process, at each level only low pass coefficients (approximation coefficients) are further decomposed by passing it through matched low pass filter and high pass filters, whereas in case of a discrete wavelet packet transform, both the high pass (detail) and low pass coefficients at each level are decomposed. [9]. A wavelet packet transform tree structure is shown in Fig. 3.

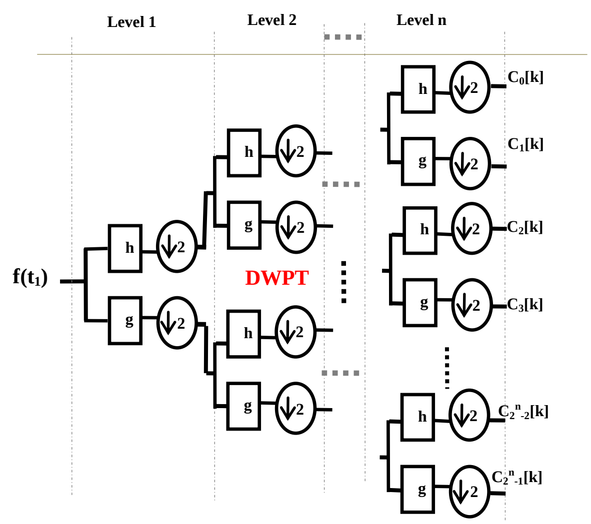


Fig. 3. Implementation of DWPT

↑2 refers to the two times up-sampling The realization of the Discrete Wavelet Packet Transform steps are mentioned as follows, The sampled time domain signal which is shown at

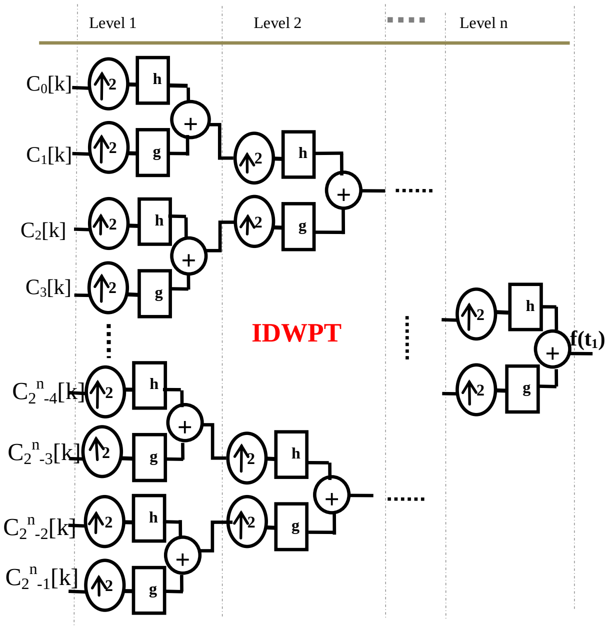


Fig. 4. Implementation of IDWPT

leftmost $f(t_1)$ in the diagram also defined as 'root' is decomposed by means of convolving the signal with high pass filter 'h' and low pass filter 'g', and then down sampled by two times. For the next level both these two outputs are again decomposed and downsampled as previous, and hence this continues to 'n' level to get the decomposed wavelet packet coefficients defined as 'leaves' shown at the rightmost in fig.3. this process of 'n' level DWPT decomposition generates a binary tree like structure as shown consists of 2^m 'high' and 'low' FIR filters (h and g) at level 'm'. [9],[12].

Wavelet packet reconstruction (WPR), is also referred as the inverse discrete wavelet packet transform (IDWPT), similar to the DWPT the IDWPT also has tree structure flow of operational steps shown in fig.4. this is exactly "mirror image" of fig.3. Here the data flow is from leftmost 'leaves' and rightmost 'root', and instead of downsampling upsampling is performed as shown in fig.4. [9],[12].

IV. Simulation Results And Analysis

The following parameters are considered for the simulation.

Table I.

Parameter	Value
M-QAM	64
No of subcarriers	64
Cyclic prefix Length	16
BCH code	(15,5,7),(15,7,5)
RS code	(15,5),(31,15),(31,23)
Convolutional code	1/2rate with CL=7

From Fig. 5. We can say that, the conventional OFDM with 1/3 rate narrowsense binary BCH code i.e.(n=15,k=5,d=7) for BER:10^{-4} provides 1dB power improvement w.r.to without coded OFDM. where for the same BER DWT based coded OFDM provides 2dB power improvement over the AWGN channel.

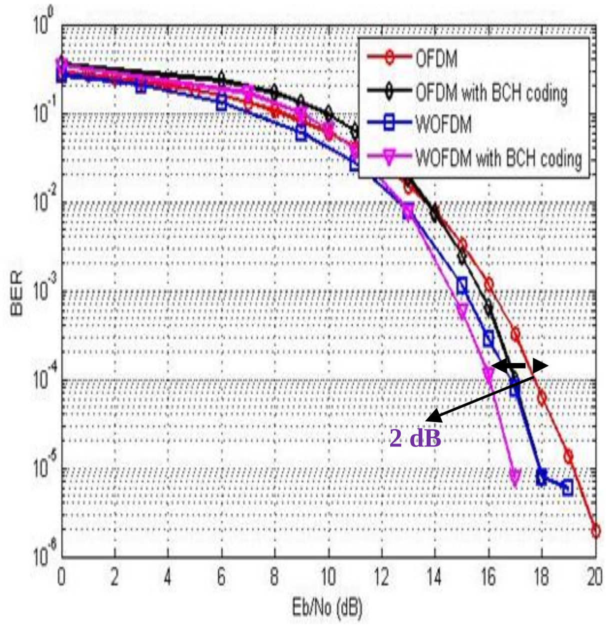


Fig. 5. BER for FFTOFDM & WOFDM over AWGN channel for (15, 5) BCH code.

From Fig. 6. We can say that, the conventional OFDM with 8 symbol error correcting 1/2 rate RS code i.e.(n=31,k=15,t=8) over GF(2^5) and for BER:10^{-3} provides 1.5dB power improvement w.r.to without coded OFDM, whereas for the same BER DWT based coded OFDM provides 2.5dB power improvement.

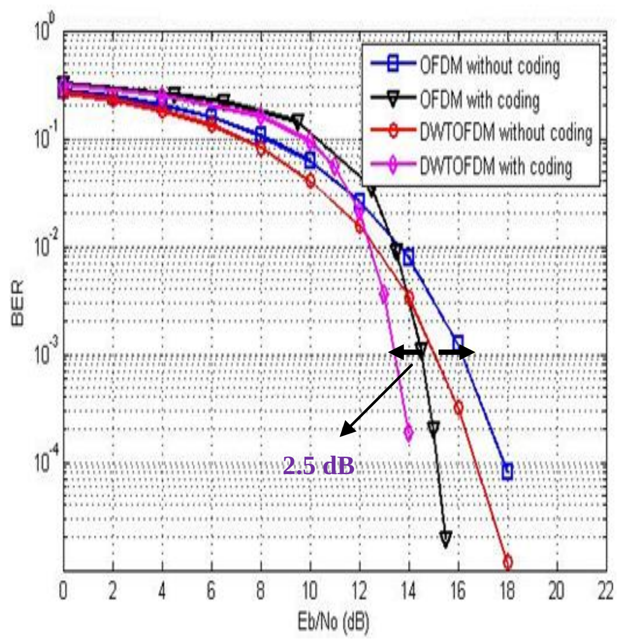


Fig. 6. BER for FFTOFDM & WOFDM over AWGN channel for (31, 15) RS code.

From Fig. 5 and Fig. 6 we can say that the 1/2 rate RS WOFDM gives more coding gain than 1/3 rate BCH coded W OFDM.

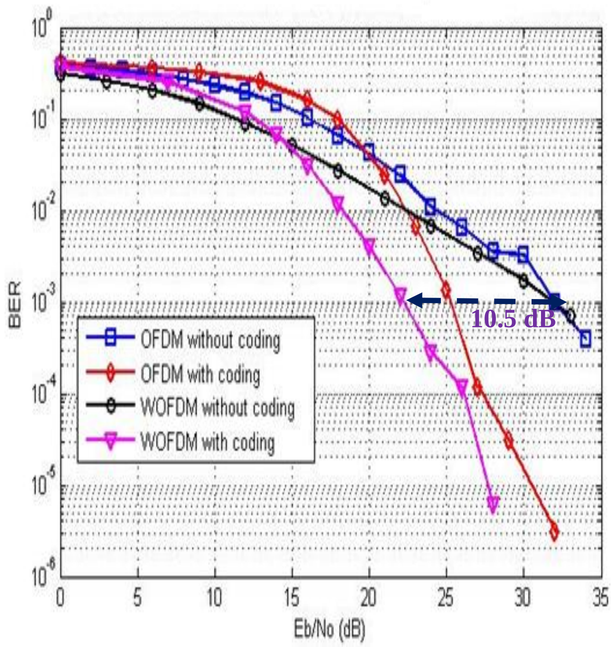


Fig. 7. BER for FFTOFDM & WOFDM over Rayleigh fading channel for (15, 7) BCH code.

Table II.For BCH and RS coding (ref.fig.5 and fig.6)

BER	Eb/No(dB)	Code rate	Coding gain	Ofdm type
10^{-4}	17	1/3	1dB	FFT
10^{-4}	16	1/3	2dB	DWT
10^{-3}	15	1/2	1dB	FFT
10^{-3}	13.5	1/2	2.5dB	DWT

From Fig.7. We can say that, the conventional OFDM with 1/2 rate narrowsense binary BCH code i.e.(n=15,k=7,d=5) for BER:10^{-3} provides 7.5dB power improvement w.r.to without coded OFDM. where for the same BER DWT based coded OFDM provides 10.5dB power improvement over the Rayleigh fading channel

From Fig. 8. We can say that, the conventional OFDM with 1/3 rate narrowsense binary BCH code i.e.(n=15,k=5,d=7) for BER: 10^{-3} provides 8.5dB power improvement w.r.to without coded OFDM. Where for the same BER DWT based coded OFDM provides 12dB power improvement over the Rayleigh fading channel.

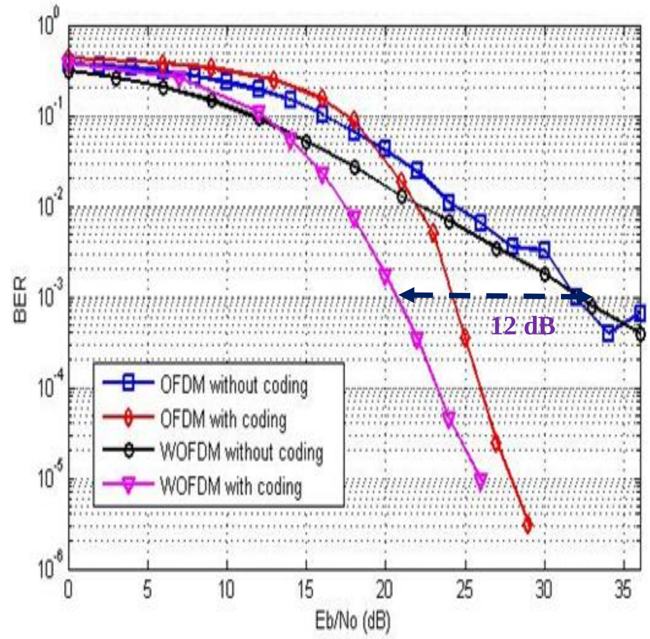


Fig. 8. BER for FFTOFDM & WOFDM over Rayleigh fading channel for (15, 5) BCH code.

Table II. For BCH coding (ref.fig.7 and fig.8)

BER	Eb/No(dB)	Code rate	Coding gain	Ofdm type
10^{-3}	25	1/2	7.5dB	FFT
10^{-3}	22	1/2	10.5dB	DWT
10^{-3}	24	1/3	8.5dB	FFT
10^{-3}	20.5	1/3	12dB	DWT

From Fig.9. We can say that the, conventional OFDM with 1/3 rate 5 symbol error correcting RS code i.e.(n=15,k=5,d=11) over GF(2^4) and for BER:10^{-3} provides 8.5dB power improvement w.r.to without coded OFDM. Where for the same BER DWT based coded OFDM provides 12.5dB power improvement over the Rayleigh fading channel.

From Fig.10. We can say that, the conventional OFDM with 8 symbol error correcting 1/2 rate RS code i.e.(n=31,k=15,t=8) over GF(2^5) and at BER:10^{-3} provides 9.5dB power improvement w.r.to without coded OFDM. Where for the same BER DWT based coded OFDM provides 13dB power improvement.

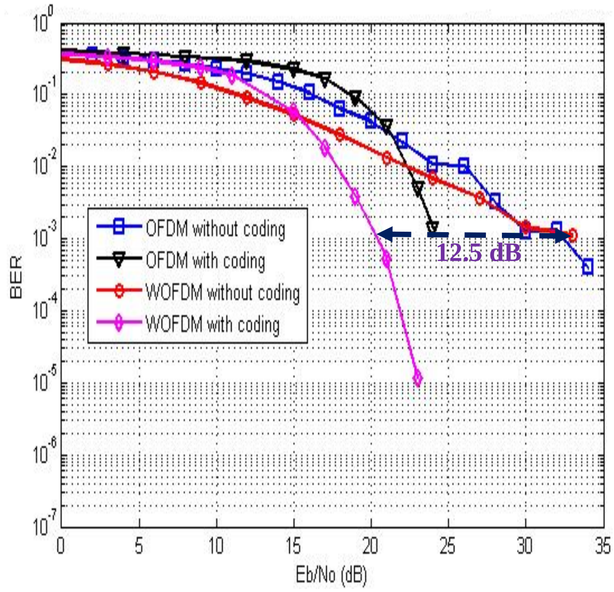


Fig. 9. BER for FFTOFDM & WOFDM over Rayleigh fading channel for (15, 5) RS code.

From Fig.11. We can say that, the conventional OFDM with 4 symbol error correcting RS code i.e.(n=31,k=23,t=4) over GF(2^5) and for BER:10^{-3} provides 8.5dB power improvement w.r.to without coded OFDM, where for the same BER DWT based coded OFDM provides 10.5dB power improvement.

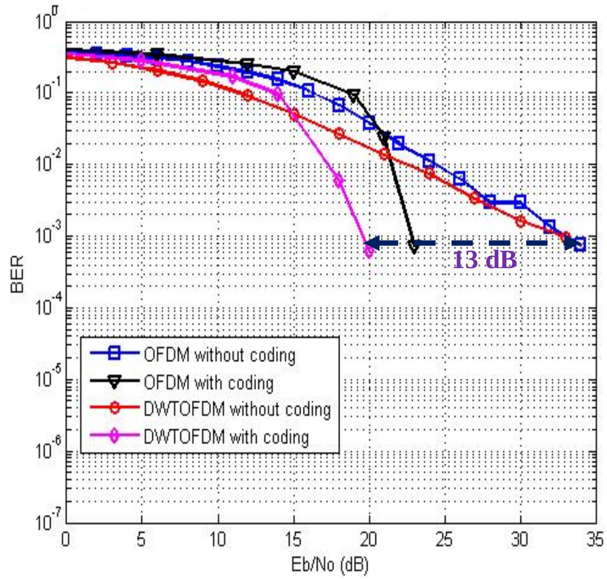


Fig. 10. BER for FFTOFDM & WOFDM over Rayleigh fading channel for (31,15) RS code over GF(2^5).

Table III. For RS coding (ref.fig.9,fig.10 and fig.11)

BER	Eb/No(dB)	t=error correction capability	Coding gain	OFDM type
10^{-3}	24	5	8.5dB	FFT
10^{-3}	20	5	12.5dB	DWT
10^{-3}	23	8	9.5dB	FFT
10^{-3}	19.5	8	13dB	DWT
10^{-3}	24	4	8.5dB	FFT
10^{-3}	22	4	10.5dB	DWT

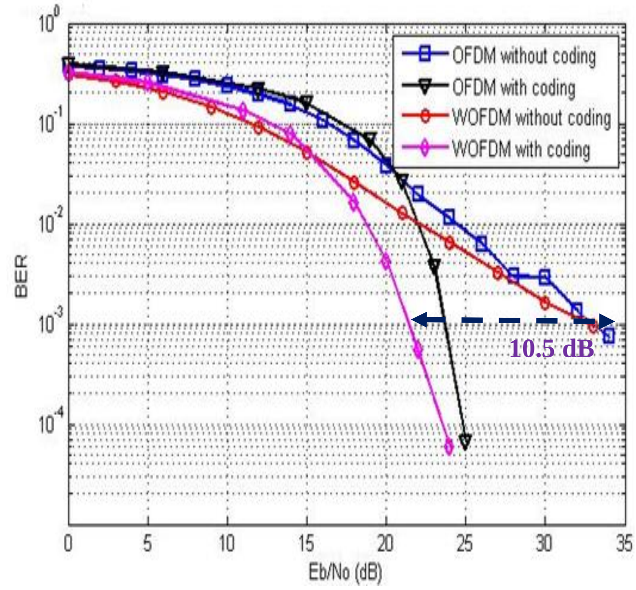


Fig. 11. BER for FFTOFDM & WOFDM over Rayleigh fading channel for (31,23) RS Code over GF(2^5).

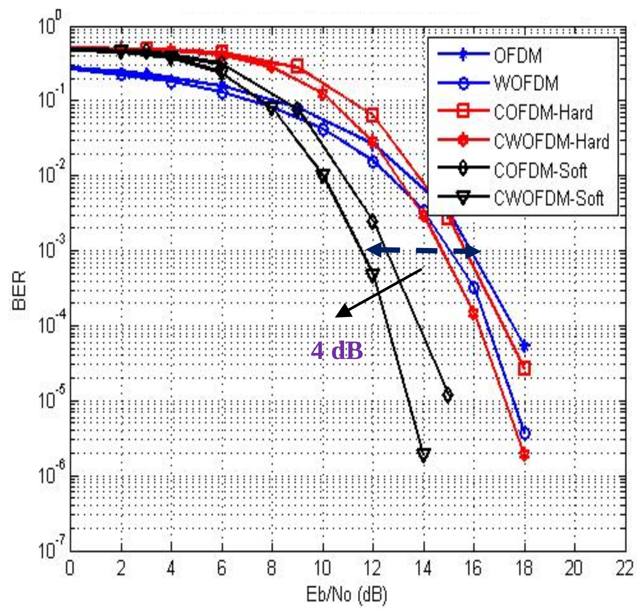


Fig. 12. BER for FFTOFDM and WOFDM over AWGN channel using 1/2 rate convolutional code with Viterbi decoding for CL=7.

Fig. 12. Gives performance comparison of conventional OFDM with WOFDM over AWGN channel using 1/2 rate convolutional code with hard and soft Viterbi decoding for constraint length=7. From the Fig.12. We can say that 1/2 rate

convolutional code with hard Viterbi decoding at BER:10^{-3} provides 0.1dB power improvement w.r.to uncoded OFDM, whereas WOFDM with hard decoding gives 1dB coding gain w.r.to uncoded conventional OFDM, and for soft decoding performance improves to 4dB.

Fig.13. Gives performance comparison of conventional OFDM with WOFDM over Rayleigh channel using 1/2 rate convolutional code with soft Viterbi decoding for constraint Length =7. From the Fig. 13. We can say that at BER: 10^{-3} WOFDM provides 5.5dB power improvement w.r.to without coded OFDM.

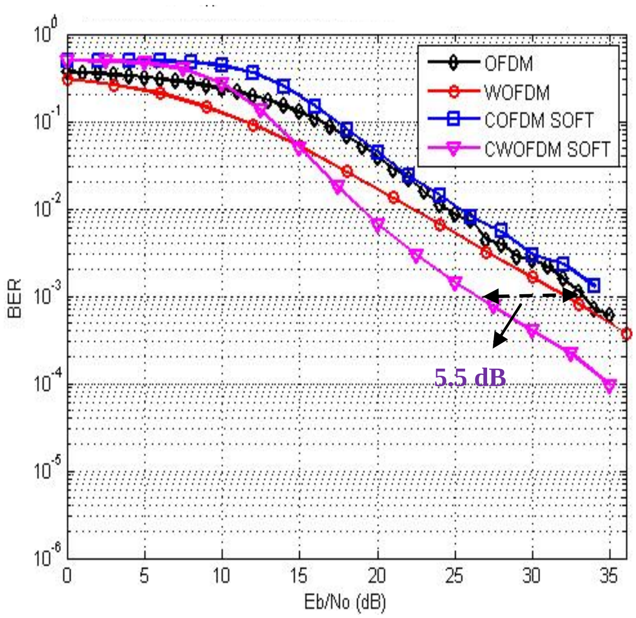


Fig. 13. BER for FFTOFDM and WOFDM over Rayleigh channel using 1/2 rate convolutional code with Viterbi decoding for CL=7.

From fig.5,6 and 12 we can say that, for lower value of SNR over AWGN channel convolutional coding gives better performance than BCH and RS ,but for Rayleigh channel RS outperforms than BCH and convolutional codes.

V. Conclusion

Wavelet based OFDM provides improved BER performance in AWGN and Rayleigh fading channel than FFT-OFDM also it is more bandwidth efficient than conventional OFDM because cyclic prefix is not added for WOFDM. In this work Haar wavelet is used for the analysis, for other wavelets performance is different, and with error control coding performance further enhanced, 1/2 rate RS coding gives more coding gain than 1/3 rate BCH code, and WOFDM with RS code outperformance than BCH and convolutional codes for Rayleigh channel.

References

[1] R W Chang, "Synthesis of band-limited orthogonal signals for multi-channel data transmission," Bell systems technical journal, Vol. 45, Dec 1966.

[2] S. B. Weinstein, P. M. Ebert, "Data transmission by frequency division multiplexing using the discrete Fourier transform," IEEE Transactions on communications, Oct 1971.

[3] Yong Soo Cho,Chung-Ang University, Republic of Korea,Jaekwon Kim,Yonsei, Won Young Yang,Chung-Ang University, Republic of Korea,Chung G. Kangkorea university, "MIMO-OFDM Wireless Communications with Matlab",IEEE Press,John Wiley & Sons (Asia) Pte Ltd.

[4] R.Asif, R.A Abd-Alhameed, O.OAnoh,YAS Dama, H.S.M igdadi and J.M.Noars, "Performance Comparison between DWT-OFDM and FFT-OFDM using time domain zero forcing equalization ," International Conference on Telecommunications and Multimedia (TEMU), 2012.

[5] Deepak Gupta, Torry Harris, Vipin B Vats Member IEEE, Waltham USA,Kamal K. Garg, "Performance analysis of DFTOFDM, DCTOFDM, and DWTOFDM systems in AWGN Channel," 2008 IEEE.

[6] Fuqin Xiong, "Digital Modulation Techniques", second Edition, © 2006 ARTECH HOUSE, INC.

[7] R. Dilmaghani, M. Ghavami "Comparison between wavelet based and Fourier based multicarrier UWB systems" ,IET Communication,2008.

[8] Scott L. Linfoot "Wavelet Families For Orthogonal Wavelet Division Multiplex", The Institution of Engineering and Technology. June 2008.

[9] An Li, William Shieh, Member, IEEE, Rodney S. Tucker, Fellow, IEEE "Wavelet Packet Transform Based OFDM for Optical Communications" , Journal of Lightwave Technology, December 2010.

[10] Ingrid Daubechies Rutgers University and AT&T Bell Laboratories, "Ten Lectures on Wavelets", society for industrial and applied mathematics, Philadelphia Pennsylvania 1992.

[11] S.G.Mallat "A Theory for Multiresolution Signal Decomposition: The Wavelet Representation", IEEE TRANSACTIONS ON PATTERN ANALYSIS AND MACHINE INTELLIGENCE,JULY 1989.

[12] O.Daoud "Performance Iimprovement of Wavelet Packet Transform over Fast Fourier Transform in Multiple-Input Multiple-Output Orthogonal Frequency Division Multiplexing Systems", IET Communication, 2012.

[13] S.L.Linfoot, M. K. Ibrahim, Marwan M. Al-Akaidi "Orthogonal Wavelet Division Multiplex-An Alternative to OFDM", IEEE Transactions on Consumer Electronics, MAY 2007..

[14] Shu Lin, Daniel J. Costello, University of California, Davis, University of Hawaii, Manoa, University of Norte Dame, "Error Control Coding" Second Edition (C)2011 PEARSON Publication.

[15] T.K.Moon, Utah State University, "Error Correction Coding Mathematical methods and algorithms" John wiley & Sons, Inc. Pub.

[16] http://www.hdplc.org/modules/about/faq.html, dated 26/03/2014

[17] https://standards.ieee.org/news/2011/bpl.html) dated ,26/03/2014.

[18] Andrew j. Viterbi, SENIOR MEMBER, IEEE, "Convolutional Codes and Their Performance in Communication Systems", IEEE TRANSACTIONS ON COMMUNICATIONS TECHNOLOGY, OCTOBER 1971.

[19] Tri T. Ha, Naval Postgraduate School, Monterey, California, "Theory and Design of Digital Communication Systems" Cambridge University Press 2011.

[20] Bernard Sklar, "Digital Communications Funadamentals and Applications" Second Edition, Pearson Education Asia 2001.