Analysis of Design Parameters in Flexible Reuse Deployments of OFDMA Downlink Cellular Networks

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Abstract—Increasing the capacity of wireless cellular networks has always been an important objective. To achieve high capacity, full frequency reuse is considered in 5G systems by the research community. However, the efficient Inter Cell Interference (ICI) mitigation methods are required since full reuse deployment leads to the problem of ICI in cellular networks. To improve the user performance, a flexible Fractional Frequency Reuse (FFR) has been emerged as a solution in recent times by overcoming the ICI. However, FFR has three important design parameters, such as Signal to Interference plus Noise Ratio (SINR) threshold, bandwidth ratio and power ratio. The optimal choice and configuration of the design parameters is very important as their configuration determines network performance. Hence, it appears highly desirable to investigate the impact of the parameters on the system performance. This paper investigates the impact of the design parameters on the deployment of flexible reuse and suggests the optimal range of parameters where the capacity of flexible reuse is increased significantly when compared against full reuse.

I. INTRODUCTION

To meet the increasing future traffic demand, the International Telecommunication Union-Radiocommunication sector (ITU-R) defined requirements, framework and overall objectives of the future development of International Mobile Telecommunications (IMT) for next generation wireless networks [1], [2]. One of the important requirements is to provide uniform services to the users irrespective of their locations, amongst others. There are certain Physical (PHY) and Medium Access Control (MAC) layer techniques being used by 4G systems in order to meet the capacity requirements. To meet the increasing capacity demands in future cellular systems, the PHY and MAC layer techniques are not sufficient. Therefore, the strategies such as reduced cell size, relays, heterogeneous architecture with micro, femto, pico cells, and frequency reuse, amongst others are being considered to increase cell capacity. Frequency reuse plan is required in future cellular systems. The reuse plan may be at the choice of the network operator. However, 4G mobile networks use OFDMA as the transmission scheme [3], and these networks considered single frequency reuse. Due to heavy co-channel interference cell edge users suffer from high outage probability. Fractional Frequency Reuse (FFR) [4] is suggested as a scheme to improve the situation and may be considered for deployment in future cellular networks. In FFR, the total available bandwidth is divided into cell center and cell edge parts as shown in Fig. 1. The frequency band $f_1$ is allocated to center users and $1/3$rd of the rest is allocated to cell edge users. The $f_2$, $f_3$ and $f_4$ indicate frequencies used in the edge regions as shown in figure. Users with good SINR condition are allocated resource from center band while users who experience poor SINR are allocated the edge band resources. FFR deployment requires three important design parameters. Firstly, the SINR threshold $\gamma_{th}$, which categorizes a user as cell center or cell edge. Secondly, the power ratio $\rho_p$ which distributes the total transmit power between both the regions. It balances the power in center band and edge band, while keeping total power constant. The power ratio influences the SINR experienced by the user in both the bands. The third parameter, by which the total frequency resource (bandwidth) is divided into cell center and cell edge bands, is bandwidth partitioning ratio $\alpha$. Bandwidth is divided into center and edge bands and that users are allotted resources according to their center and edge class. SINR threshold $\gamma_{th}$ which is used to classify users plays a vital role in user throughput. A low $\gamma_{th}$ would put more users in center band while a high value will do the reverse. The resource requirement of a user is dependent on the SINR which is divided by power ratio $\rho_p$. A combination of these parameters thus influences the performance of such networks. Hence, the aim of this work is to find optimum range of the values of the design parameters $\gamma_{th}$, $\alpha$ and $\rho_p$ in case of FFR under UMi scenario for which system capacity, in terms of

![Fig. 1. Flexible FFR with power configuration](image-url)
number of users, is improved when compared with that of full reuse scheme (reuse one) for Real Time (RT) traffic.

A. Related Work

Authors in [4], [5] studied FFR scheme with different power levels over the cell center and edge bands. To classify the users into center and edge, this work assumes a fixed SINR metric, and the performance of FFR has not been described for optimal range of SINR thresholds. The work in [6] discussed the impact of scheduling strategies and number of users in a cell but it has not analyzed FFR for different SINR thresholds and power ratios, and has not shown the improvement on cell edge performance. The authors in [7] have presented the evaluation of FFR scheme, where the base station distribution following Poisson point process with limited usability. Fixed partitioning of the available bandwidth is used by those authors for both center and edge band users. Fixed thresholds are used in terms of distance and SINR. The expected performance gains of FFR with irregular cell patterns are examined in [8], [9] by considering fixed thresholds in terms of distance and SINR for distinguishing users into cell center and cell edge.

The existing available literature on FFR focuses on Best Effort (BE) traffic and evaluate the performance of FFR for an arbitrary value of the important design parameters $\gamma_{th}$, $\alpha$ and $\rho_p$. All those works in literature presented results only for BE traffic. However, a suitable and optimal choice of the values of the parameters in FFR scheme which provides notable gain in overall cell performance is still to be addressed. Since the interference generated is heavily dependent on the amount of transmit power, the load and frequency reuse factor, it is important to analyze the effect of those parameters for FFR in order to find the configuration for which the overall cell capacity performance is improved for regular (hexagonal) and irregular grid (realistic) layout as well. However, in this paper we investigate the performance for regular grid only.

Although it is found that there is some amount of work on the performance evaluation for BE traffic for FFR, there is hardly any attention paid to the performance evaluation of RT traffic. However, there are some works carried out on the performance analysis of frequency reuse planning systems for RT service recently [10], [11] while serving RT and BE traffic by considering equal and unequal power levels over the bands. A huge amount of RT traffic is carried in OFDMA traffic by considering equal and unequal power levels over the cell center and edge bands. To classify the traffic for RT service recently [10], [11] while serving RT and BE traffic, a huge amount of RT traffic is carried in OFDMA traffic by considering equal and unequal power levels over the cell center and edge bands. To classify the traffic for RT service recently [10], [11] while serving RT and BE traffic, a huge amount of RT traffic is carried in OFDMA traffic by considering equal and unequal power levels over the cell center and edge bands.

The interference term in the denominator of equation 1 includes the total interference power from the desired base station. The set $\{b\}$ is the index of base stations which cause interference in $b^{th}$ band.

1) Channel model: The interference term in the denominator of equation 1 includes the total interference power received from center and edge frequency bands from all the base stations. Therefore, the total interference power in $b^{th}$ band for a user is given as

$$P_f(b) = \sum_{i \in I_c} P_t(c, L) d_{u,i}^{-\gamma_p} \cdot \chi_{u,i} |h_{u,i}|^2 + \sum_{i \in I_e} P_t(e, L) d_{u,i}^{-\gamma_p} \cdot \chi_{u,i} |h_{u,i}|^2,$$

where $I_c$ consists of all interfering base stations transmitting in the center band with power $P_t(c)$. Similarly, $I_e$ consists of all interfering base stations transmitting in edge band with power $P_t(e)$. $\chi_{u,i}$ is the shadowing component which is log normal distributed, $n_p$ is the pathloss exponent, $\rho_p$ is power ratio and $L$ includes fixed loss.

The distance from $i^{th}$ base station to a user ‘$u$’ is $d_{u,i}$ as seen form Fig. 1. From equation 3, it is assumed that the transmission power and shadowing component is assumed to be identical for all the base stations. The transmit power from $i^{th}$ base station is $P_t$. From equation 1, desired signal power and individual interference powers are assumed to follow lognormal distribution while evaluating SINR distribution. The received total interference power from all the base stations at a point is approximated as a lognormal random variable (RV) using F-W method [15]. The channel powers from the desired and interfered base stations are modeled as lognormal RV having mean $\mu_{sh-Rayleigh}(r, \theta)$ and variance $\sigma_{sh-Rayleigh}^2(r, \theta) = \zeta^2 \sigma^2_{h} + 5.57^2$ [16], where $\zeta = 0.1 \times \ln 10$ is a scaling constant. Since we consider the Rayleigh distribution for fast fading $h$, the power of fast fading $|h|^2$ follows Gamma distribution with unity mean.
2) Power configuration in FFR: Let the total bandwidth $\mathbb{B}$ is divided between the center and edge band in the ratio, $\alpha$. The bandwidth allotted to cell center region is $B_c = \alpha \mathbb{B}$ and the amount of bandwidth allotted to cell edge region is $B_e = (1-\alpha) \mathbb{B}$ as shown in Fig. 1. Let the total transmit power from the downlink transmitting antenna be $P_T$. We define $\rho_{pc}$ to be the power per Hz in center region and $\rho_{pe}$ to be the power per Hz in edge region. Therefore, the power ratio $\rho_p$ is defined as the power spectral density of cell center region to power spectral density of cell edge region, that is, $\rho_p = \frac{\rho_{pc}}{\rho_{pe}}$.

Therefore, in FFR we write

$$P_T = \rho_{pc}(\alpha \mathbb{B}) + \rho_{pe}(\frac{1-\alpha}{3})\mathbb{B},$$

(4)

the total transmit power for center and edge band region is $P_{T_c} = \rho_{pc}(\alpha \mathbb{B})$ and $P_{T_e} = \rho_{pe}(\frac{1-\alpha}{3})\mathbb{B}$ such that $P_{T_c} + P_{T_e} = P_T$. Using the $\rho_p$, the power spectral density over the center and edge bands is expressed as

$$\rho_{pc} = \frac{3P_T \rho_p}{1 + \alpha(3\rho_p - 1)}\mathbb{B},$$

(5)

and

$$\rho_{pe} = \frac{3P_T}{1 + (3\rho_p - 1)}\mathbb{B}.$$  

(6)

Hence, by using these relations, the total transmit power for center band is derived as

$$P_{T_c} = \frac{3P_T \rho_p \alpha}{1 + \alpha(3\rho_p - 1)}\mathbb{B},$$

(7)

and total transmit power for edge band as

$$P_{T_e} = \frac{P_T(1 - \alpha)}{1 + \alpha(3\rho_p - 1)}\mathbb{B}.$$  

(8)

A user is allocated to band $b = \text{‘}c\text{‘}$ (center band) if $\gamma_{u,c}(r,\theta) \geq \gamma_{th}$, otherwise a user is allocated to band $b = \text{‘}e\text{‘}$ (edge band), where $\gamma_{u,c}(r,\theta)$ is the mean SINR of a user at a location when in center band, $\gamma_{th}$ is the SINR threshold and is another design parameter. The effective bandwidth required to guarantee delivery of a RT service with an equivalent bit rate requirement of $R_u$ by considering semi persistent scheduling of RT traffic while fulfilling the QoS requirements (delay) is as given in [11]. In RT service, packet delay is an important QoS measure of performance. Hence, the issues related to packet delay during a call are addressed by the Packet Scheduling- Radio Resource Allocation (PS-RRA) unit in the base station as in [12], [17]. Let the total number of subcarriers available be $N_{sc}$. Subcarrier bandwidth $\Delta f_{sc} = \frac{\mathbb{B}}{N_{sc}}$, $f_s$ is the oversampling factor.

The average number of required carriers needed to make a call as a function of area averaged SINR to finally evaluate cell capacity (in next section) [10] are given as

$$N_{avg,b}(\gamma) = \sum_{k=1}^{K_b} N_{k,b} \int_{\gamma_{th}}^{\gamma_{k+1}} p(\gamma) d\gamma,$$  

(9)

where $p(\gamma)$ is the probability density function (PDF) of area averaged SINR. The average number of subcarriers required by the cell center and edge users to make a call is dependent on $\sinr{10}$. In FFR, the number of available carriers for center and edge band users can be expressed as $N_{sc,c} = \left\lfloor \frac{\alpha \mathbb{B}}{3} \right\rfloor$ and $N_{sc,e} = \left\lfloor \frac{1-\alpha}{3} \mathbb{B} \right\rfloor$ respectively. The supported number of channels (simultaneous calls) for a given effective rate requirement, $R_u$ in a band ‘$b$’ is

$$N_{ch,b} = \left\lceil \frac{N_{sc,b}}{R_u} \right\rceil,$$  

(10)

Therefore the total number of channels is

$$N_{ch} = N_{ch,c} + N_{ch,e}.$$  

(11)

III. BANDWIDTH PARTITIONING: PROBABILITY BASED METHOD OF SELECTING THE BANDS

Let $N_u$ be the total number of users deployed in the cell. Number of users in a band is $N_{u,b} = N_u P_{A_b}$, where $P_{A_b}$ is the area averaged probability of selecting band $b$. The probability of a user ‘$u$’ at a location $(r, \theta)$, being in CB is given by

$$P_{bc}(\gamma_{u,c}(r,\theta) > \gamma_{th}) = \frac{1}{2} - \frac{1}{2} \text{erf} \left( \frac{\gamma_{th} - \gamma_{u,c}(r,\theta)}{\sigma \gamma \sqrt{2}} \right),$$  

(12)

where $\gamma_{u,c}(r,\theta)$ and $\sigma$ are the mean and standard deviation of the SINR, $\gamma_{u,c}(r,\theta)$ is the center band user SINR at the location $(r, \theta)$. The probability of a user to be in cell center or edge region is found based on their SINR condition using equation 1. Therefore, the area averaged probability of selecting center band $P_{A_c}$ is given by

$$P_{A_c} = \int_{r, \theta} \left( \frac{1}{2} - \frac{1}{2} \text{erf} \left( \frac{\gamma_{th} - \gamma_{u,c}(r,\theta)}{\sigma \gamma \sqrt{2}} \right) \right) p_u(r,\theta) r d\theta,$$  

(13)

where $p_u(r,\theta)=r/(\pi R^2)$ for uniform user distribution. Therefore, the approach to partition the bandwidth in a cell is by finding the probability of a user to be in center or edge band. That is, it is based on the area averaged probability of selecting the bands. The ratio of bandwidth allotted to center band users is equal to the probability of an user to be at that band. So, the bandwidth ratio is defined as $\alpha = P_{A_c}$. Hence, area averaged probability of selecting the edge band is given by $P_{A_e} = 1 - P_{A_c}$.

Fig. 2. Probability of selecting the bands with design parameters

The probability of selecting center and edge bands as given by the relation 13, when $\gamma_{th}$ and $\rho_p$ change, can be seen from Fig. 2. As $\gamma_{th}$ increases the probability of selecting the edge band is high and vice versa. It means that the percentage of
users to be in edge band is more. Fig. 3 shows the number of channels in center and edge band vs. $\gamma_{th}$ and $\rho_p$. It is evaluated by using the relation 10. From our analysis, we say that $\alpha$ is function of $\gamma_{th}$. As $\gamma_{th}$ increases $\alpha$ decreases as shown in figure (Fig. 5 of next section). This implies that the numerator of the relation 10 is affected by $\gamma_{th}$. This is seen from Fig. 3. As $\gamma_{th}$ increases, the number of channels in edge band increases. This is because when $\gamma_{th}$ increases the range of SINR $\gamma$ in edge band is more. This leads to more percentage of situations with larger number of carriers required to make a call. This affects the denominator of relation 10. Together it yields a region of $\gamma_{th}$ where number of channels is greater than the reference as shown in Fig. 4. In reference scheme, the available number of channels is 125, almost constant, at all $\gamma_{th}$ and $\rho_p$. However, for FFR, it is seen that when $\gamma_{th}$ is from -4 dB to 14 dB, the number of channels in FFR is more (at $\gamma_{th} = 6$ dB) than that of the reuse one. In addition to this, $\rho_p$ also affects the number of channels.

![Fig. 3. Number of channels in center and edge bands with design parameters](image-url)

As it is considered that the bandwidth to be allocated is such that the bit rate requirement is to be guaranteed to a user as per [10], the capacity is evaluated in terms of number of users. Therefore the objective of this work is stated as to find the values of design parameters $\gamma_{th}$, $\rho_p$ and $\alpha$ so that the system capacity in terms of number of users $N_u$ is maximized for RT traffic. This can be expressed as

$$\left(\gamma_{th}, \alpha^+, \rho_p^+\right) = \arg\max_{\gamma_{th}, \alpha, \rho_p} [N_u],$$  \quad (14)

subject to the following constraints:

- $0 \leq \gamma_{th} \leq \gamma_{max}$
- $0 \leq \alpha \leq 1$
- $0 \leq \rho_p \leq 1$

From the above, the number of users supported in a cell in FFR based partitioning is given as

$$N_u = \left[\frac{\alpha B}{B_{A_e}}\right] + \left[\frac{1 - \alpha}{3}\right] \frac{B}{B_{A_e}},$$  \quad (15)

where $\alpha = P_{A_e}$ and number of users in center and edge band is given as $N_{u,c} = \left[\frac{\alpha B}{B_{A_e}}\right]$ and $N_{u,e} = \left[\frac{(1 - \alpha) B}{B_{A_e}}\right]$, where the average bandwidth required by a user in center band ($B_{A_e}$) and edge band ($B_{A_e}$) [10].

Therefore the total number of users is $N_u = N_{u,c} + N_{u,e}$. From the analytical framework and the results above, we say that as $\rho_p$ increases, the number of channels in center band increases and number of channels in edge band decreases, and vice versa. When $\gamma_{th}$ is very low, the number of channels in edge is minimum. However, proper selection of $\gamma_{th}$ and $\rho_p$ is required in order to study the behavior of RT traffic since these values influence the capacity. Hence this gives the trade off and optimization of traffic capacity as a function of $\alpha$, $\gamma_{th}$ and $\rho_p$.

### IV. Analytical Results and Discussions

In this section, the results generated for the above bandwidth partitioning method are described. The performance results are provided for UMi scenario. The simulation parameters used for performance evaluation are given in Table I. It is assumed that the rate requirement of a user is 12.2 Kbps for VoIP in this work. The performance is compared against the full reuse (reference scheme). The Fig. 5 shows the relationship amongst $\gamma_{th}$, $\alpha$ and $\rho_p$ design parameters for FFR based partitioning. The 3-D plot is obtained for UMi scenario. When $\gamma_{th}$ increases $\alpha$ decreases. The reason being that when $\gamma_{th}$ varies the percentage of users between center and edge band changes as discussed in previous section. That is, when $\gamma_{th}$ increases the probability of a user to be in edge band increases, hence larger portion of the bandwidth is allotted to edge band.
region and smaller portion to center band. This can be better explained with the readings shown in Fig. 5. At $\gamma_{th} = -5$ dB, the bandwidth ratio $\alpha$ is equal to 0.99 which is almost equal to 1. It means that 99% of the bandwidth is allotted to center users as all users are switching to center region. At these values of $\gamma_{th}$ and $\alpha$, it is seen that the $p_p$ is 0.69. At this $p_p$, the powers transmitted over center and edge bands is calculated using the power formulations 7 and 8. The center power is 40.06 dBm (10.14 Watts) and edge power is 33.89 dBm (2.36 watts). However, at $\gamma_{th}$ of 22 dB, the $\alpha = 0.06$. That is, percentage of users in edge band is more which is a reverse behaviour of the lower $\gamma_{th}$ region. When $\gamma_{th}$ changes from low to high values the $\alpha$ changes which can be seen at different $p_p$. Four sets of the values of the parameters shown in the plot are provided in Table II.

<table>
<thead>
<tr>
<th>$\gamma_{th}$</th>
<th>$p_p$</th>
<th>$\alpha$</th>
<th>full reuse</th>
<th>FFR</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5 dB</td>
<td>0.69</td>
<td>0.99</td>
<td>125</td>
<td>125</td>
</tr>
<tr>
<td>0 dB</td>
<td>0.0001</td>
<td>0.63</td>
<td>125</td>
<td>139</td>
</tr>
<tr>
<td>6 dB</td>
<td>0.66</td>
<td>0.43</td>
<td>125</td>
<td>173</td>
</tr>
<tr>
<td>22 dB</td>
<td>0.33</td>
<td>0.06</td>
<td>125</td>
<td>105</td>
</tr>
</tbody>
</table>

From the above, we say that $p_p$ and $\gamma_{th}$ influence the number of channels and the probability of selecting center or edge band users in a cell in FFR based partitioning. However, an optimum selection of $\gamma_{th}$, $\alpha$ and $p_p$ is required in order to study the behavior of RT traffic since these values depend on the system capacity. Further, it is pertinent to note that performance gains of the scheme will be different if the realistic layout is considered. Therefore, the optimum selection of the design parameters is more important while choosing the reuse plans for deployment since they influence the performance gains, and the choice is left to the system designer.

V. CONCLUSION

A framework of important design parameters in FFR OFDMA networks by considering regular grid layout for RT traffic is presented in this paper. For RT traffic, with proper choice of SINR threshold $\gamma_{th}$, bandwidth ratio $\alpha$ and power ratio $p_p$ parameters, the capacity of FFR is improved significantly over the reference. From the results, it is found that with appropriate and optimal choice of the values of design parameters, FFR provides a notable gain in overall cell performance over the reference. Hence, the scheme can be used for OFDMA based cellular networks for improving cell capacity, but with carefully chosen values of design parameters. The objective of choosing the design parameters is based on the system designers requirement. Moreover, it is highly desirable to investigate the impact of the schemes by considering the realistic cellular grid layout which is taken up a future work item. The proposed work is flexible to be adapted to realistic cellular grids.

REFERENCES


Fig. 5. 3D plot of design parameters in flexible FFR

Fig. 6. Number of users supportable in flexible FFR and full reuse

This is the key result which gives the relationship amongst three system design parameters based on which the network performance depends. The number of supportable users is shown in Fig. 6. The figure shows for different $\gamma_{th}$, $p_p$ and $\alpha$ values. At both lower and higher $\gamma_{th}$, the number of users supported is less. This happens at all $p_p$. However, the capacity is higher from the range of values from 0 to 10 dB of $\gamma_{th}$. It is observed that at very low $p_p$ (0.0001), the capacity drops at all $\gamma_{th}$. This is due to fact that when $p_p$ is very low, the power transmit to edge band is more, and hence may lead to a slightly drop in capacity. But, in the FFR based partitioning scheme, the capacity is better than reference scheme from 0 to 10 dB of $\gamma_{th}$. From this analysis, it is observed that, at $\gamma_{th} = 6$ dB the number of users supportable is approximately equal to 173 where $p_p = 0.66$. This is observed at $\alpha = 0.439$. However, at lower and higher $\gamma_{th}$ the capacity goes down, and the values are indicated and can be seen from Fig. 6. The number of users supportable in reference scheme is 125 as seen from Fig. 6. But, at the lowest power ratios, say at $p_p=0.0001$, there is a sudden drop in the capacity of reference scheme. This is due to fact that at the lowest power ratios ($p_p = 0$), there is hardly no transmission over the band (as reuse one is a single band scheme), which will lead to sudden drop in capacity of reuse one. Hence, the FFR performance is better and the gain is improved by 38% over the reference.