Analysis of Total Harmonic Modulation Distortion of a Directly Modulated and Optically Injected Fabry-Perot Laser Diode

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***Abstract*—This paper presents calculations on total harmonic modulation distortion level of an injection-locked Fabry-Perot laser diode directly modulated by a sinusoidal modulating signal accompanied by the harmonics of the fundamental modulation. The theory predicts that the total harmonic distortion level shows a resonance like effect followed by a fall-off with modulation frequency. The novel prediction of the theory is that optical injection favors total harmonic modulation distortion suppression and the lower the optical injection level, the better is the suppression. This is due to the fact that when relative optical injection power is small, the lockband is also narrow. So, the higher order modulation sidebands fall outside the lockband and get suppressed. This makes the output THD power level lower.**

Keywords— THD; Optical injection-locking; Modulation; Linewidth enhancement factor; Fabry-Perot Laser Diode;

I. INTRODUCTION

Optical communication in the 21st century is going to be the dominant mode of electromagnetic communication. This is because light as a carrier of information has a latent bandwidth of the order of tens of THz. 1550 nm is chosen as the telecommunication wavelength because of low loss of optical fiber on one hand and minimum dispersion obtained through zero dispersion wavelength shift of single mode optical fiber on the other hand.

One factor of major concern in telecommunication is the modulation distortion which can be internal as well as external to the laser diode (LD). Here, we consider external modulation distortion [1] where the pre-modulation message signal itself contains harmonic distortion. No research seems to be carried out in this regard yet.

Our objective is to calculate the total harmonic modulation distortion at the output of the injection-locked laser diode which is directly modulated by the distorted message signal and injected by a coherent CW light. It is expected that optical injection locking of the directly modulated LD will result in a reduction of modulation distortion level. The dependence of total harmonic modulation distortion level suppression upon light injection on various laser parameters and locking parameters can be predicted from this analysis.

One important application of this study is in ultradense wavelength division multiplexed passive optical network (UDWDM PON) [2 – 14]. If harmonic distortion be a matter of concern in UDWDM-PON, then the harmonic distortion can be reduced through optical injection by making a proper choice of system parameters.

II. ANALYSIS

To analyze the effect of coherent light injection into a directly modulated Fabry-Perot laser diode (FPLD), we make use of transmission line model [15] of the injection-locked FPLD developed by the author. We consider multi-harmonic contaminated sinusoidal modulation. This harmonically distorted modulation can arise from the process of generation and conversion of the message signal.

Let the output lightwave of the directly-modulated slave LD be written as:

$$E_{\text{mod}}(t) = E_{0f}(t)e^{j(\omega_0 t + \theta_n(t))} \tag{1}$$

where,

$$E_{of}(t) = E_f(1 + \sum_{n=1}^{N} m_{an} \sin n\omega_m t) \tag{2}$$

Here E_f is the electric field amplitude, m_{an} is the amplitude modulation (AM) index of the n-th harmonic of the fundamental modulating signal while ω_m is the angular frequency of the fundamental modulating signal. ω_0 is the angular frequency of the free-running FPLD. N is the number of harmonics present in the message signal. $\theta_n(t)$ is the angle modulation term for the n-th harmonic generated through direct modulation.

The CW lightwave injected from the master FPLD will have an electric field given by:

$$E_{in}(t) = E_{in0} e^{j(\omega_{in} t + \theta_{in})} \tag{3}$$

where E_{in0} is the electric field amplitude of the CW lightwave injected into the slave laser diode, ω_{in} is the input light angular frequency and θ_{in} is an arbitrary phase angle.

The electric field of the locked FPLD output lightwave can be written as :

$$E_{Ln}(t) = E_{n0} e^{j(\omega_{in} t + \theta_{0n}(t))} \tag{4}$$

From the equation [15 – 20] governing the behaviour of the injected slave laser diode, we can derive the amplitude and phase governing equations of the injected slave LD by applying the Principle of Harmonic Balance [21, 22]. The equations can be expressed as:

$$\frac{2Q}{\omega_0}\frac{1}{|E_{n0}(t)|}\frac{d|E_{n0}(t)|}{dt} = -2 + \left[C_1 - C_2\frac{|E_{n0}(t)|^2}{|E_{0f}(t)|^2}\right]\left[\frac{|E_{in}(t)|}{|E_{n0}(t)|}\cos\left(\theta_{in} - \theta_{0n}(t)\right) + 1\right] \tag{5}$$

and

$$\frac{2Q}{\omega_0}\frac{d\theta_{0n}(t)}{dt} = \left[C_1 - C_2\frac{|E_{n0}(t)|^2}{|E_{0f}(t)|^2}\right]\left[\frac{|E_{in}(t)|}{|E_{n0}(t)|}\sin\left(\theta_{in} - \theta_{0n}(t)\right)\right] - 2\delta Q - \alpha\frac{|E_{in}(t)|}{|E_{n0}(t)|}\cos\left(\theta_{in} - \theta_{0n}(t)\right) \tag{6}$$

Equations (5) and (6) have been derived using low level optical injection, i.e., the injected optical power, P_i being much less than the free-running output optical power P_m of the slave FPLD.

C_1 and C_2 are laser constants which depend upon laser parameters including the characteristics of the active region of the LD.

$[\theta_{in} - \theta_{0n}(t)]$ is the modulated input-output phase error of the locked slave LD corresponding to n-th harmonic modulation. α is the linewidth enhancement factor [23 – 31]. which is also known as phase-amplitude coupling factor of the FPLD. $\delta = \frac{\omega_{in} - \omega_0}{\omega_0}$ is the normalized detuning of the input lightwave from the free-running slave LD. ω_0 is the free-running angular frequency of the slave FPLD in absence of injection.

Equation (5) is known as the "Amplitude Equation" and (6) is called the "Phase Equation" of the injection-locked Fabry-Perot laser diode. Here,

$$C_1 = (\mu - \frac{1}{\mu})\ell\alpha_m \tag{7}$$

$$C_2 = (\mu - \frac{1}{\mu})\ell g_0 \frac{|E_f|^2}{|E_s|^2} \tag{8}$$

$$Q=(\mu-\frac{1}{\mu})\frac{2\pi\ell}{\lambda} \qquad (9)$$

where ℓ is the LD cavity length, μ is the refractive index of the active medium of the LD, α_m is the mirror loss, g_0 is the linear gain of the LD, Q is the external Q-factor of the FPLD, E_f is the electric field amplitude of the free-running slave LD, E_s is the saturation electric field of the slave LD, and. λ is the operating wavelength of the slave FPLD.

The solutions for amplitude and phase equation of the locked slave LD are assumed in the form:

$$E_{n0}(t)=E_f\left[1+\sum_{n=1}^{N}m_{0n}\sin(n\omega_m t+\xi_n)\right] \qquad (10)$$

and

$$\theta_{0n}(t)=\theta_{avg}+\sum_{n=1}^{N}\theta_n^{'}\sin(n\omega_m t+\xi_n) \qquad (11)$$

where m_{0n} is the output AM index and $\theta_n^{'}$ is the output phase modulation amplitude for the n-th harmonic modulation. θ_{avg} is the average output phase. Carrying out some mathematical manipulation with amplitude equation we get:

$$\frac{m_{0n}^2}{m_{an}^2}=\frac{\left[z_3\frac{\theta_n}{m_{an}}+z_4\right]^2}{n^2z_1^2+z_2^2} \qquad (12)$$

where,

$$z_1=\frac{\omega_m}{\omega_0}Q \qquad (13)$$

$$z_2=C_2+\frac{1}{2}(C_1+C_2)G\cos\zeta \qquad (14)$$

$$z_3=(C_1-C_2)G\sin\zeta \qquad (15)$$

$$z_4=C_2[1+G\cos\zeta] \qquad (16)$$

where G is the locking amplitude gain, $\zeta=\theta_{in}-\theta_{avg}=$ input-output dc phase error and $G^2={P_i}\big/{P_m}$. Detailed nonlinear analysis of the phase equation leads to the following equation:

$$\frac{\theta_n^2}{m_{an}^2}=\frac{(w_3-w_4)^2+n^2w_1^2\,{\alpha^2}\big/{4}}{n^2w_1^2+w_2^2} \qquad (17)$$

where,

$$w_1=\frac{2Q\omega_m}{\omega_0} \qquad (18)$$

$$w_2=G[(C_1-C_2)\cos\zeta+\alpha\sin\zeta] \qquad (19)$$

$$w_3=\frac{G}{2}\left[(C_1+C_2)\sin\zeta-\alpha\cos\zeta\right] \qquad (20)$$

$$w_4=C_2G\cos\zeta \qquad (21)$$

The total harmonic modulation distortion of the injected LD in optical domain is calculated as:

$$THD\big|_{Optical}=\frac{\sum_{n=2}^{N}m_{0n}^2}{\sum_{n=1}^{N}m_{0n}^2}=\frac{\sum_{n=2}^{N}m_{0n}^2}{m_{01}^2+\sum_{n=2}^{N}m_{0n}^2}$$
$$=\left[\frac{\sum_{n=2}^{N}\frac{\left(z_3\frac{\theta_n}{m_{an}}+z_4\right)^2}{n^2z_1^2+z_2^2}}{\frac{\left(z_3\frac{\theta_1}{m_{a1}}+z_4\right)^2}{z_1^2+z_2^2}}\frac{m_{an}^2}{m_{a1}^2}\right]-\left[\frac{\sum_{n=2}^{N}\frac{\left(z_3\frac{\theta_n}{m_{an}}+z_4\right)^2}{n^2z_1^2+z_2^2}}{\frac{\left(z_3\frac{\theta_1}{m_{a1}}+z_4\right)^2}{z_1^2+z_2^2}}\frac{m_{an}^2}{m_{a1}^2}\right]^2 \qquad (22)$$

The THD at the output in the electrical domain is given by:

$$THD\big|_{Electrical}=THD\big|_{Optical}^2 \qquad (23)$$

We have taken a reasonable value of the linewidth enhancement factor, $\alpha = 8$, in conformity with our measured value [30] of α for the same FPLD. We have taken $\delta = 0$ in the numerical calculation as a specific case. However, other values of δ are also possible subject to the condition that injection locking is not disturbed.

The calculated values for different modulation frequencies are given in the following Table I, Input THD = -20 dBc:

TABLE I.

Serial number	Modulation frequency (GHz)	Output THD for Pi/Pm = - 10 dB	Output THD for Pi/Pm = - 15 dB	Output THD for Pi/Pm = - 20 dB
1.	0.1	-24.96	-25.44	-25.43
2.	0.5	-22.11	-22.33	-22.68
3.	1	-19.38	-20.04	-21.53
4.	1.5	-17.78	-19.25	-21.32
5.	2	-16.91	-18.93	-21.5
6.	2.5	-16.47	-18.92	-21.87
7.	3	-16.27	-19.08	-22.37
8.	5	-16.74	-20.57	-24.83
9.	7	-18.01	-22.54	-27.39
10.	9	-19.43	-24.48	-29.76
11.	10	-20.34	-25.55	-30.58

a. Input THD = -20 dBc

The calculated values of output total harmonic modulation distortion for three different optical injection power level have been plotted in Fig. 1 as a function of fundamental modulation frequency. The THD at the modulation input is taken at -20 dBc throughout for numerical calculation.

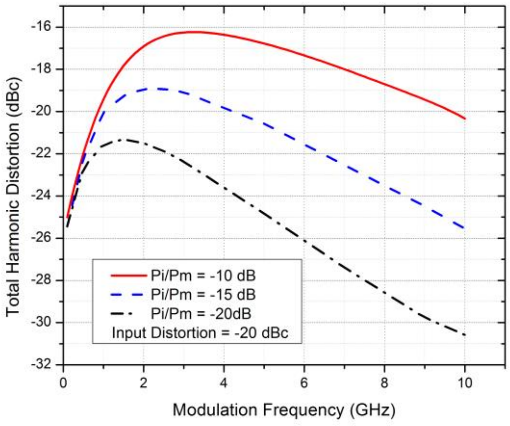

Figure 1: Total harmonic modulation distortion in dBc as a function of fundamental modulation frequency using the relative optical injection power level as a parameter. $\alpha = 8, \delta = 0,$ Input THD = -20 dBc

Fig. 1 implies that there is always a suppression of THD at the output of injection-locked FPLD due to optical injection. The THD suppression is more pronounced at higher modulation frequencies. The plot shows a resonance-like phenomenon, the THD suppression attaining a minimum level (corresponding to the peaks in the plot) at certain modulation frequency. The THD suppression increases (i.e., the resonance curve falls off) rapidly beyond the peaks in the plot. At higher levels of optical injection, the resonance becomes less pronounced. These are novel observations predicted by our analysis.

III. RESULTS AND CONCLUSION

The detailed analysis of the locked LD shows that suppression of modulation distortion takes place in all cases upon coherent light injection. The analysis is based on our transmission line model [15] of the Fabry-Perot laser diode which transforms the laser diode into an optoelectronic circuit and theory makes it possible to analyse the phenomenon of optical injection locking. It is also noticed that the linewidth enhancement factor of the semiconductor laser plays a significant role in estimating the total harmonic modulation distortion reduction upon external CW light injection. The degree of suppression depends upon the power of injected light relative to the free-running slave laser diode, presence of other harmonic distortion terms at the input, magnitude of lockband and also upon the inherent nonlinearity of the LD. The THD as a function of modulation frequency shows a resonance like phenomenon which is sensitive to the optical injection level. The lower the optical injection power level the faster is the THD fall off with modulation frequency from resonance peak. This behaviour is due to the fact that when the injection power level is low, the lockband becomes smaller. As a result, the modulation sidebands fall outside the lockband and get suppressed. This, in turn, gives rise to a fall in THD level with lowering of the relative injection power.

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