

# Gear Geometry Analysis with Asymmetric Pressure Angle

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**Abstract :** Gear design is one of the most critical components in the Mechanical Power Transmission industry. Among all the gear

design parameters pressure angle is the most critical parameter, which mainly affects the load carrying capacity of the gear. Generally gears are designed with a symmetric pressure angle for drive and coast side. It means that both flank side of gear are able to have same load carrying capacity. In some applications, such as in wind turbines, the gears experience only unidirectional loading. In such cases, the geometry of the drive side need not be symmetric to the coast side. This allows for the design of gears with asymmetric teeth. Therefore new gear designs are needed because of the increasing performance requirements, such as high load capacity, high endurance, long life, and high speed. These gears provide flexibility to designers due to their non-standard design. If they are correctly designed, they can make important contributions to the improvement of designs in aerospace industry, automobile industry, and wind turbine industry.

In this paper we present a mathematical model of helical gear pair with an asymmetric pressure angle. We have increased the pressure angle of gear on the drive side to increase the load capacity and performance of the gear pair in terms of noise and mesh stiffness while transmitting power. Also we have analyzed the results of bending and contact stresses generated on gear pair with the asymmetric pressure angle.

## I. INTRODUCTION

Gear is a part which is used to transmit the power which that transmits rotational force from one gear to another. Depending upon the application the construction, design & arrangement, these device can be transmit power or forces at different torques & speeds in different way(direction) from power source. When gear is designed by considering teeth profiles so as to get or a produce a constant angular velocity ratio during meshing, this action are said to be Combination function.

The most usually used combination tooth profile is the a involute profile because it can be manufacture very easily and the centre distance between one gear to another gear of velocity ratio can be verify without changing their setup. In traditional involute helical and spur gears, both sides (drive and coast) flanks perception the same contact strength & bending. However, in some of the cases for applications in automotive field coast side and drive side are not equally loaded while transmitting force or power, the coast side gear tooth is small as compared to the drive side which is loaded for longer period of time.

Hertzian contact stress is a description of the stress within mating parts. The Hertzian contact stress usually refers to

the stress near or close to the area of contact between two gear teeth is may be point or line contact. Also call as Hertzian stress. Hertzian stress is based on the compressive load normal to the geometry of contact surfaces at the contact zone. Gear failure can be happened by one of this mode Contact or hertzian fatigue failure. The tooth surface undergoes fluctuating, repeated and cyclic stresses of all kinds during the course of this action, fatigue failure of the tooth surface is likely. The failure of the tooth surface due to fatigue is termed as 'Pitting'. Contact stress in meshing gears depends upon the geometry of mating teeth, which is not constant along the path of contact. The teeth geometry also depends on gear module and pressure angle. The present work is related to study of influence of module and pressure angle on geometric features of mating teeth which in turn affects the contact stresses induced and the contact fatigue life. To study the effect of above parameters ANSYS was used. A spur gear pair with asymmetric influence of pressure angle is shown in Fig.1.

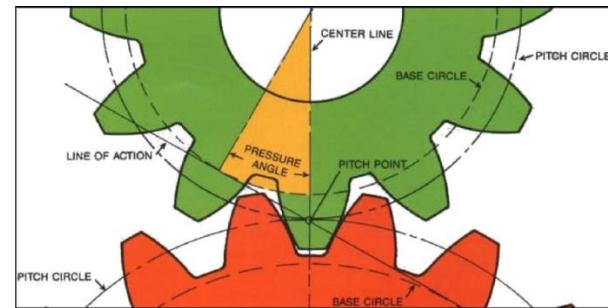


Fig. 1. Asymmetric Influence of module of Pressure Angle Gear Pair

## 2. Literature Review

- Gear design by A.J. Lemanski - analysis of gear capacity by means of tooth strength and allowable stress.
- Finite element modelling spur and helical gears in contact by David J. Sell - provides information on bending stress, load distribution and transmission error.

Designing highly loaded spur and helical gears for truck transmission that are both strong and quite requires an analysis method that can easily be implemented and also provides information on bending stress, load distribution and transmission error. In order to reduce the modelling time, a pre-processor program that creates the geometry needed for a finite element analysis has been developed. While requiring a minimum of user input, the program generates a 3-dimensional model of contracting spur or helical gears using eight node brick element. Gap elements are used to model the contact that may occur that normally occurs between meshing gear teeth as well as the contact that may occur of

the line of action due the teeth deflecting under load. The effects of nonstandard geometry such as profile modification and tooth errors can be easily included in the model. The processor creates an input files used by a commercially available finite element code. The technique used to develop the pre-processor program is presented.

Gear design is discussed in terms of drive system requirements; preliminary design considerations; analysis of gear capacity by means of tooth strength rating and allowable stress index; computer design analysis; design optimization and kinematic action analysis by computer and special considerations.

This paper presents direct gear design – an alternative method of analysis and design of involute gears which separates gear geometry definition from tool selection to achieve the best possible performance for particular product and application. This method has successfully been applied for a number of automotive applications.

The method/approach of reverse engineering provides an approximate condition of the gear pair by determining the key basic parameters such as Pressure angle, Module, Helix angle, Correction Factor, Measurement Over Ball/Pin, Tip Circle Diameter etc. and these parameters are further used for detail design or as a reference for new design.

### 3. Influence of Pressure angle

The important thing in selection of pressure angle of drive side& coast side teeth for an asymmetric gear design purpose. Although the increase in pressure angle will increase the bending strength, there is a limit to the increase in pressure angle because it reduces the top land thickness of tooth which makes the tooth sharp at top which is known as ‘Tooth Peaking’. Also increasing the pressure angle decreases the contact and overlap ratio which affects the performance in terms of NVH. By increasing the pressure angle the sliding velocity of the gear reduces because of increase in flank curvature, this in turn reduces the noise and vibration and increases the stiffness of the gear tooth in loaded condition.

An asymmetric gear tooth design effects the following terms and factors related to gear geometry which will be discussed in detail.

1. Tooth Top land thickness
2. Contact Ratio
3. Sliding velocity
4. Bending strength
5. Contact Strength

#### 3.1 Tooth top land thickness

A major issue or limitation with increasing the pressure angle is the reduction of gear tooth width at the addendum circle. Tooth shape becomes more and more pointed as the pressure angle increases and the top land becomes smaller which results in permanent deflection at the tooth tip. This phenomenon is termed as “Tooth Peaking”. The peaking limit sets the limit of the pressure angle. The effect on the tooth flank and top land thickness with the increase in pressure angle is shown in Fig. 2

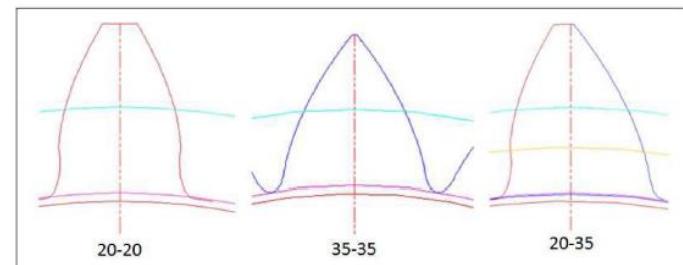


Fig. 2. Change in Tooth Flank and Top land Thickness w.r.t Pressure Angle

#### 3.2 Contact Ratio

Contact ratio is the measure of the average number of teeth in contact along the total path of contact. It means the contact period in which a tooth comes and goes in and out of contact with the mating gear. As the pressure angle increases the contact ratio reduces and load increases on gear tooth. In general the contact ratio of a gear pair should be a minimum 1.10, below this limit the loading on gear tooth increases, which results in tooth breakage during cyclic loading. Also the highest point of single tooth contact (HPSTC) reaches near the tooth tip causes permanent deflection in gear teeth and affects the gear performance in terms of noise and vibration. Hence it is important to the keep maximum pressure angle on drive side so as to maintain a minimum contact ratio of 1.10 and keeping same pressure angle on coast side for better performance.

#### 3.3 Sliding Velocity

Gear tooth sliding velocity is defined as a difference between rolling velocities of teeth in mesh. For two involute profile teeth in mesh, at a given point on the line of action, a product of radius of curvature and the rotational speed are not equal; therefore, the resultant rolling velocities are different. The pitch point is the only contact point for the meshing teeth where there is a pure rolling. At any other location during the interaction of tooth profile there is some sliding.

#### 3.4 Gear Tooth Bending Stress

Determination of the capacity of the gear to transfer the required torque for the desired operating life is completed by determining the strength of the gear teeth in bending and durability of teeth. Gear tooth bending strength increases as the stresses at the tooth root decreases. As the pressure angle increases the tooth root thickness at the critical section increases hence reducing stress at the tooth root. Fig. 5 shows, the direction of load applied on gear tooth while transferring torque.

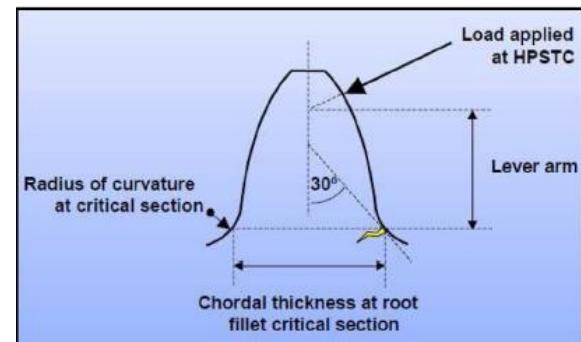


Fig. 5. Schematic representation of load applied on Gear tooth

### 3.5 Gear tooth contact strength

When a gear pair meshes, the region of contact theoretically is a line. The curvature of the individual mating surfaces at the point of contact will vary according to the position of the point of contact on the line of action, as the gear tooth surface rolls and slides during this action. Though theoretically it is a line contact, the line actually develops into the band of certain width along the length of the tooth due to mutual comprehensive pressure. As the tooth surfaces move relative to each other with a combination of rolling and sliding, this band also continuously moves. Since the tooth surface undergoes fluctuating, repeated and cyclic stresses of all kinds during the course of this action, fatigue failure of the tooth surface is likely. The failure of the tooth surface due to fatigue is termed as 'Pitting'. Contact fatigue strength of a gear material plays a major role in determining the service life of gears. It has been observed that pitting mostly occurs in the area surrounding the pitch line. This may be due to the fact that the direction of sliding velocity changes at the pitch point, resulting in the comprehensive stress increasing in that region. Heinrich Hertz, a German physicist, developed the expression for the stresses generated when two curved surface are in contact. These surface stresses are generally known as Hertzian Stresses

### 4. Geometry design

A mathematical model of an asymmetric pressure angle gear pair geometry has been designed, consideration of the key parameters of gear geometry; module, helix angle, number of teeth, face width and centre distance, and the results of the bending and contact strength is compared to a symmetric pressure angle gear pair with the same parameters. It has been observed that the bending stress at the gear root and the contact stress on gear flank are reduced in the asymmetric pressure angle gear pair hence increasing load carrying capacity approximately 20% when compared with a symmetric pressure angle gear pair. Relevant parameters shown in Table.1 are considerations when designing a symmetric and asymmetric pressure angle gear pair.

Symbol	Description	Symmetric gear design		Asymmetric gear design	
		Pini on	Ge ar	Pini on	Ge ar
<b>Z</b>	No. of teeth	19 17		19 17	
<b>mn</b>	Normal module mn		2.20		2.20
<b>αd</b>	Pressure angle drive side (degree )		20		35
<b>αc</b>	Pressure angle coast side (degree )		20		20
<b>β</b>	Helix		24		24

	angle (degree )		
<b>b</b>	Face width (mm)	20 20	20 20
<b>a</b>	Center distance (mm)	108	108
<b>T</b>	Input torque (Nm)	310	310

Table 1. Input Parameters for Gear Pair Design

Symmetric pressure angle geometry is formed by two symmetrical involutes with one base circle, but asymmetric pressure angle gear geometry is formed by two different involutes of different base circles. Symmetric gear tooth geometry with pressure angle 20° and 35°. Tooth geometry with a 20° pressure angle have sufficient tip thickness and sufficient root radius, but the thickness at critical section of tooth root is less which causes high bending stress and low load bearing capacity. Also the radius of curvature of the involute is low which increases the sliding velocity during mesh. Tooth geometry with a 30° pressure angle shown in, thickness at the critical section of the tooth root increases which reduces the bending stress and increases the load bearing capacity. Also the radius of curvature of the involute curve increases which reduces the sliding velocity hence decreases contact stress on the tooth flank. But as shown, there is not sufficient thickness at the tip which results in tip breakage or permanent deflection at the tip during torque transfer. Also there is not sufficient space between two teeth at the root which results in the tooth tip contacting the mating gear in the root radius. To increase the load bearing capacity and contact strength while maintaining sufficient thickness at the tip to prevent permanent deflection and sufficient space at the root to provide root radius clearance, a gear pair with a 35° pressure angle on drive side and a 20° pressure angle on coast side has been designed and analyzed. The analysis of a gear pair with CAE (Computer Aided Engineering) and FEA (Finite Element Analysis) in terms of bending strength, contact strength and tooth deflection during torque transfer is shown.

Asymmetric pressure angle gear geometry is verified by measuring the diameter over pin 'DOP' considering suitable diameter of pin. DOP shown in Fig. 9 is calculated by equations given below: Asymmetric pressure angle gear geometry is verified by measuring the diameter over pin 'DOP' considering suitable diameter of pin. DOP shown in Fig. 9 is calculated by equations given below:

$$DOP = dp + D$$

$$dp = dbd / \cos(\alpha pd) = dbc / \cos(\alpha pc)$$

$$\alpha pd = Sna/d + \text{inv}(\alpha d) + D/dbd - \pi/z$$

$$\alpha pc = Sna/d + \text{inv}(\alpha c) + Ddbc - \pi/z$$

$$dbd = d * \text{Cos}(\alpha d)$$

$$dbc = d * \text{Cos}(\alpha c)$$

$$\alpha d = \text{ATAN}(\text{TAN}(\alpha d) / \text{Cos} \beta)$$

$$\alpha c = \text{ATAN}(\text{TAN}(\alpha c) / \text{Cos} \beta)$$

Tip thickness or top land thickness of the tooth with an asymmetric pressure angle is calculated. Minimum tip tooth thickness as per DIN and ISO Gear Standards should be 0.4

times of module, to prevent tip breakage and permanent deflection at tip of teeth.

Total contact ratio (transverse and overlap) for an asymmetric pressure angle gear pair is calculated. The standards require the minimum contact ratio should be 1.1, but it is better to maintain a contact ratio of more than 2.0, so that the load is always shared by two teeth to reduce the contact stress on gear flank. Sliding velocity  $v_g$  of gear pair with symmetric and asymmetric pressure angle is calculated, and it is observed that as the contact ratio and length of path of contact decreases, the sliding velocity and specific sliding of gear pair decreases, reducing the noise and vibration.

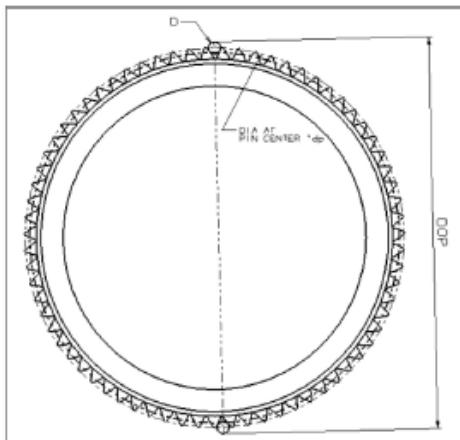


Fig. 9. Measurement Of Diameter Over Pin

## 5.FEA Analysis and Results

A mathematical model of the asymmetric gear pair with a  $35^\circ$  pressure angle on the drive side and a  $20^\circ$  pressure angle on the coast side is compared with math model of symmetric gear pair with  $20^\circ$  pressure angle on both sides. The analysis is done on CAE using NX Nastran. The bending stress, contact stress and deflection at the tooth root and tooth flank is analyzed by applying a load of 8560 N normal to tooth flank as shown in Fig.

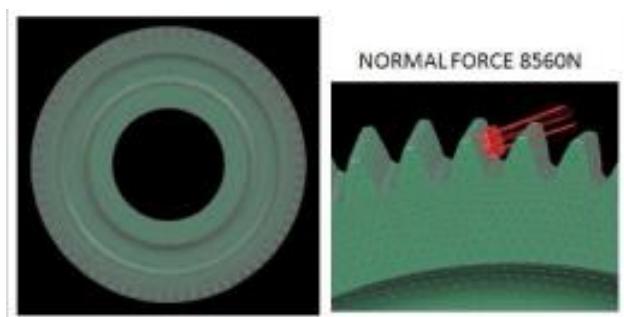


Fig. 10. Load Applied Normal To Symmetric Gear Flank Face

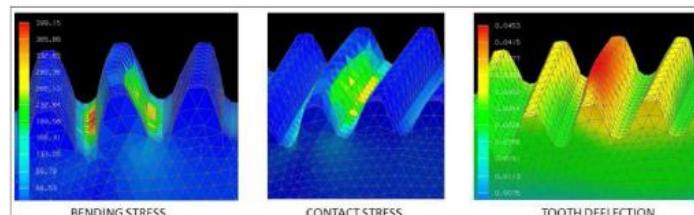


Fig.11. Tooth Stress And Deflection On Gear With Symmetric Pressure Angle (20° - 20°)

Bending stress of 391.15 MPa, contact stress of 1839.28 MPa and tooth deflection of 45 micron is found with a symmetric pressure angle gear tooth, as shown in Fig.11.

Similarly, a load of 8560N is applied normal to gear tooth with a  $35^\circ$  pressure angle on drive side and a  $20^\circ$  pressure angle on coast side. Bending stress of 306.0 MPa, contact stress of 1759.27 MPa and tooth deflection of 37.2 micron is found with the asymmetric pressure angle gear tooth, as shown in Fig.12.

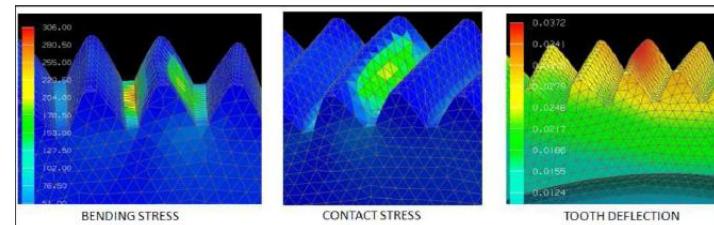


Fig 12. Tooth Stress And Deflection On Gear With Asymmetric Pressure Angle (35° - 20°)

Similarly, with a load of 8560N on the pinion tooth, bending stress of 408.39MPa, contact stress of 1839.28 Mpa and tooth deflection of 24 micron is observed with the symmetric pressure angle of  $20^\circ$ . With the same load of 8560N on the pinion tooth, bending stress of 342.70MPa, contact stress of 1759.27MPa and tooth deflection of 17 micron is observed with the asymmetric pressure angle of  $35^\circ$  on drive side and  $20^\circ$  on coast side, as shown in Fig. 13.

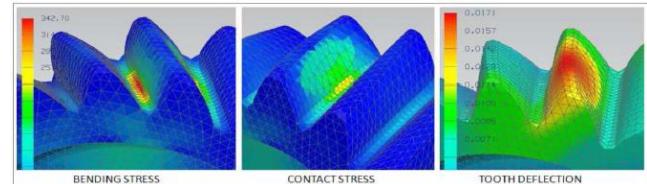


Fig 13. Tooth Stress And Deflection On Pinion With Asymmetric Pressure Angle (35° - 20°)

## 6.1 ANALYSIS OF SINGLE GEAR PAIR IN ROMAX DESIGNER:

The torque input given 3000 Nmm at the speed of 1440 rpm. The analysis of this pair is done to find out the safety factor of gear in both contact and bending and the transmission error. This gear pair is analyzed for the duration of 3 hours with different loadings at the output shaft end.

## RESULTS:

### 6.2 Gear parameter details:

Parameters	Driver	Driven
No. of teeth (Z)	58	58
face width (b)	15	15
Normal module (mn)		2
Pressure angle (phi)		20
Helix angle ( $\beta$ )		26
Center distance (c)		128
Quality Standard		ISO 7

Table 2: Test gear parameters

From the above mentioned gear parameters the gears are modeled in Romax. These gears mounted on two shafts namely driver and driven which are supported with bearing at the both ends as shown in figure. Following are the results obtained in this analysis:

### 6.3 SAFETY FACTORS IN CONTACT AND BENDING:

Gear	Contact Stress	Bending Stress	Safety Factor
	L R e i f g t h t	L R e i f g t h t	C B o e n n ta di ct n g
		(MPa)	
Driver	0 518.08	0 1 0 3. 5 4	3. 5. 6 8
Driven	0 518.08	0 1 0 3. 5 4	3. 5. 6 8

Table 3: Factor of safety in contact and bending

It has been found that the factor of safety of both driver and driven gear in both contact and bending is large enough for the analyzed duty cycle. It can be said from the above table the gears are safe.

### 6.4 ANALYZED MODEL OF TEST GEAR PAIR:

It can be seen from the above figure, the total system deflection for the single gear pair is 72.53  $\mu$ m, which is very less and in the acceptable range [14].

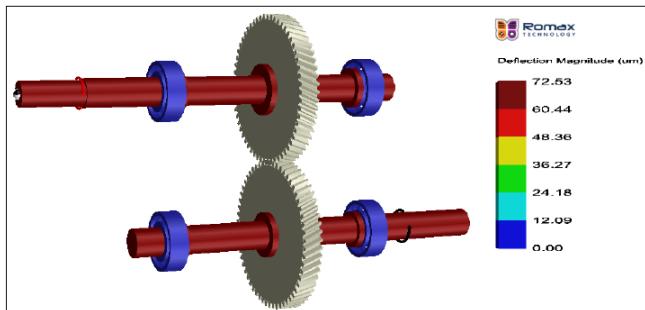


Fig 14 : Analyzed model of test gear pair

#### 7.5.4 LOAD DISTRIBUTION:

It can be seen from the above figure that the load distribution per unit length of the face width has shifted from the centre of the gear tooth to the edge of the tooth. The maximum load per unit length without applying any micro geometry is 1177.9 N/mm.

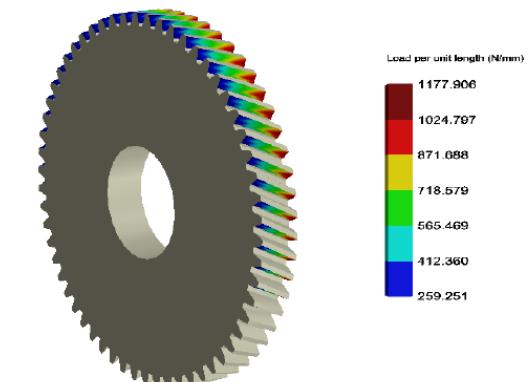


Fig 15 : Load distribution per unit length for the driver gear

### 7. Summary

As the pressure angle of a spur and helical gear pair increases, it will show improvement in the aspect of bending and contact strength, but there are limitations which limit the increase in the pressure angle desired. An asymmetric pressure angle gear design overcomes many of these limitations, resulting in better life of the gear pair with reduced weight and a decrease in sliding velocity, this result in reduced noise and vibration and an increase in the load carrying capacity of gear and an increase in durability under dynamic conditions.

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